Physical Optics

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Fourth lecture [Young's Double-Slit Exp., Fresnel Exp.]

1. Young's Double-Slit Experiment

The decisive experiment performed by Thomas Young in 1802 is shown in Fig. 1. Monochromatic light is first allowed to pass through a single small hole in an aperture in order to approximate a single point source S. the light spread out in spherical waves from the source according to Huygens' principle and is allowed to fall on the two closely spaced holes S_1 and S_2, in the aperture. The holes thus become two coherent sources of light, whose interference can be observed on a screen some distance away. If the two holes are equal in size, light emanating from the holes have comparable amplitudes, and the irradiance at any point of superposition is given by previous Eq. $I = 4\, I_0 cos^2 \left(\frac{\delta}{2}\right)$ ------------------ (1).

For observation points, such as P, on the screen distance s from the aperture, the phase difference δ between the two waves arriving must be determined to calculate the resultant irradiance there. Clearly, if $S_2P - S_1P =$ m λ, the waves will arrive in phase, and maximum irradiance or brightness results. If $S_2P - S_1P =$ (m+ ½) λ, the requisite condition for destructive interference or darkness is met. Practically speaking, the hole separation a is much smaller than the screen distance s, allowing a simple expression for the path distance, $S_2P - S_1P$. using P as a center, let an arc S_1Q be drawn of radius S_1P, so that it intersects the line $S_2\,P$ at Q. Then $S_2P - S_1P$ is equal to the segment Δ, as shown. The first approximation is to regard arc S_1Q as a straight-line segment that from one leg of right triangle, S_1S_2Q. if θ is the angle between the aperture and S_1Q, Δ= a sin θ. The second approximation identifies the angle θ with the angle between the optical axis *OX* and the line drawn from the midpoint *O* between holes to the point *P* at the screen. Observe that the corresponding sides of the two angles θ are related such that *OX* — S_1S_2, *and OP* is almost exactly perpendicular to S_1Q. The condition for constructive interference at a point *P* on the screen is then,

$$S_2P - S_1P = \Delta = m\,\lambda = a\sin\theta \quad \text{---------------------- (2)}$$

For destructive interference,

$$\Delta = \left(m + \frac{1}{2}\right)\lambda = a\sin\theta \quad \text{----------------------------- (3)}$$

Where *m* is zero of integral value. The irradiance on the screen, at a point determined by the angle θ, is found using Eq. (1) and the relationship between path difference Δ and phase difference δ.

$$\delta = \left(\frac{2\pi}{\lambda}\right)\Delta \quad \text{-------------------------------- (4)}$$

The result is

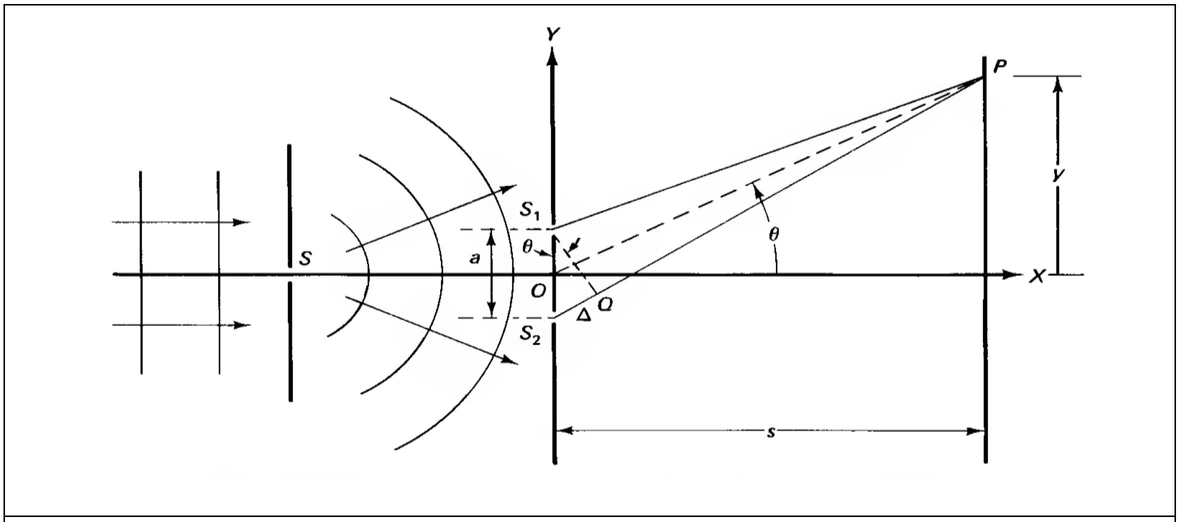

Fig. 1 Schematic for Young's Exp. The holes S_1 and $S_2$are usually slits.

$$I = 4\,I_0\,cos^2\left(\frac{\pi\Delta}{\lambda}\right) = 4\,I_0 cos^2\left(\frac{\pi\,a\sin\theta}{\lambda}\right)$$

For points *P* near the optical axis, where $y \ll s$, we may approximate further,

$$I = 4\,I_0\,cos^2\left(\frac{\pi a y}{\lambda s}\right) \text{ ------------------------- (5)}$$

From Eq. (2) for bright fringe positions in the form

$$y = \frac{m\lambda s}{a}\ , m = 0,1,2,3,\ldots, \text{ -------------------------------- (6)}$$

We fined a constant separation between irradiance maxima, corresponding to successive values of *m* , given by

$$\Delta y = \frac{\lambda s}{a} \text{ -------------------------------- (7)}$$

With minima situated midway between them. *Thus, fringe separation is proportional both to wavelength and screen distance and inversely proportional to the hole spacing.* Reducing the hole spacing expands the fringe pattern formed by each color. Measurement of the fringe separation provides a means of determining the wavelength of light. The single hole may be eliminated if laser light, both highly monochromatic and spatial coherent, is used to illuminated the double slit. As indicated in Fig. 2, fringe maxima coincide with integral orders of *m*, and fringe minima fall halfway between maxima.

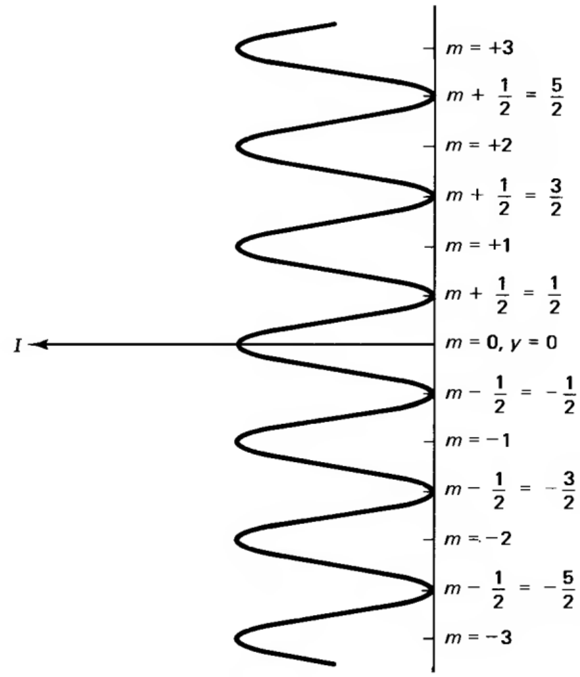

Fig. 2 Irradiance versus distance from optical axis for double slit fringe pattern.

Example

Light from a narrow slit passes through two identical and parallel slits, 0.2 mm apart. Interference fringes are seen on a screen 1 m away, with a separation of 3.29 mm. How does the irradiance at the screen vary, if the contribution of one slit alone is I_0? What is the wavelength of the light?

From Eq. (7)

$$\lambda = \frac{a\,\Delta y}{s} = (0.0002)\,(3.29\times 10^{-3})/1 = 658nm$$

According to Eq. (5), in this case

$$I = 4I_0 cos^2\,[\pi\,(0.0002)y/\,(658\times 10^{-9})(1)] = 4I_0\,cos^2\,(955y)$$

An alternative way to view the formation of bright (B) positions of constructive interference and dark (D) positions of destructive interference is shown in Fig. 3.

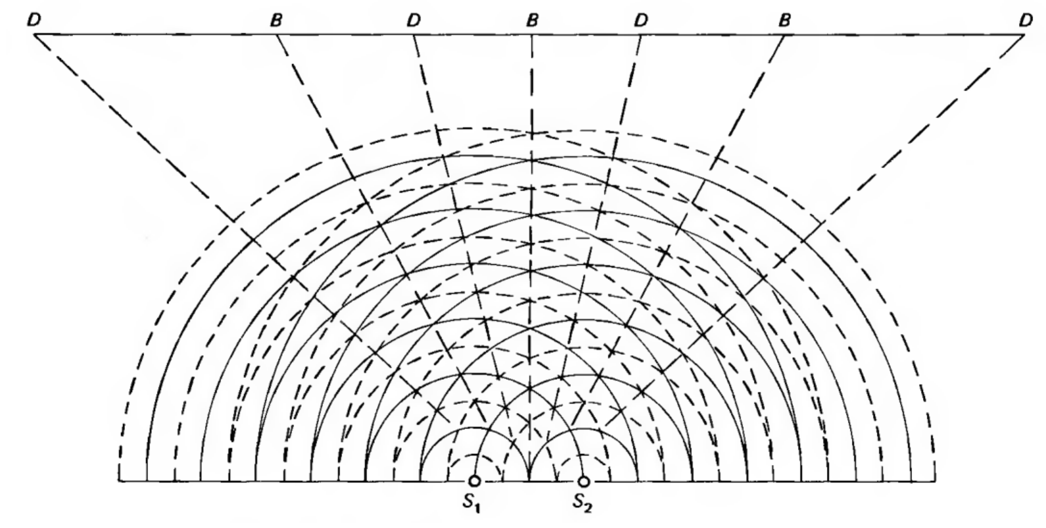

Fig. 3 alternating bright and dark interference fringes are produced by light from two coherent sources.

2. Fresnel's Biprism

Soon after the double-slit experiment was performed by Young, the objection was raised that the bright fringes he observed were probably due to some complicated modification of the light by the edges of the slits and not to true interference. Thus, the wave theory of light was still questioned. Not many years passed, however, before Fresnel brought forward several new experiments in which the interference of two beams of light was proved in a manner not open to the above objection. One of these, the Fresnel biprism experiment, will be described in some detail. A schematic diagram of the biprism experiment is shown in Fig. 4. The thin double prism *P* refracts the light from the slit sources S into two overlapping beams *ae* and *be.* If screens *M* and *N* are placed as shown in the figure, interference fringes are observed only in the region *bc.* When the screen *ae* is replaced by a photographic plate, a picture like the upper one in Fig. 5 is obtained. The closely spaced fringes in the center of the photograph are due to interference, while the wide fringes at the edge of the pattern are due to diffraction. When the screens *M* and *N* are removed from the light path, the two beams will overlap over the whole region *ae.* The lower photograph in Fig. 5 shows for this case the equally spaced interference fringes superimposed on the diffraction pattern of a wide aperture. With such an experiment Fresnel was able to produce interference without relying upon diffraction to bring the interfering beams together.

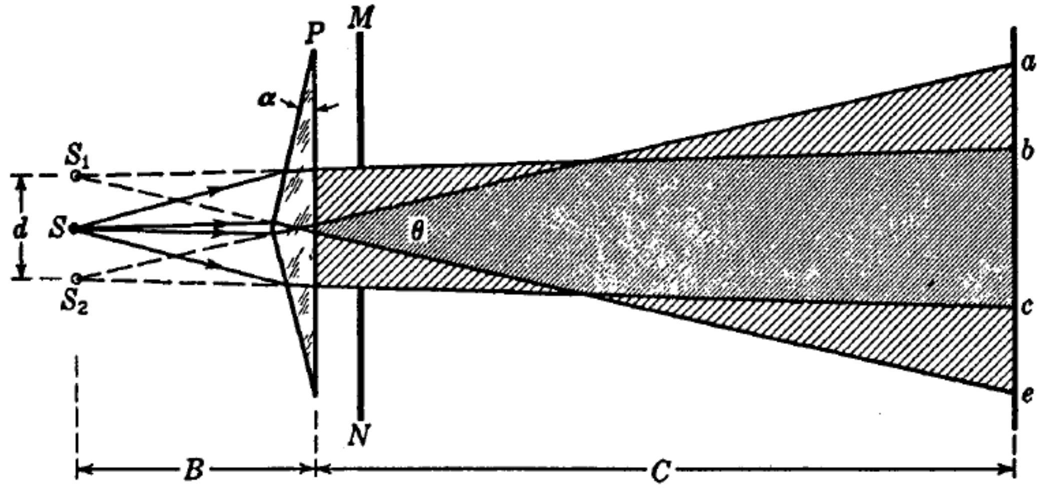

Fig. 4 Diagram of Fresnel's biprism experiment.

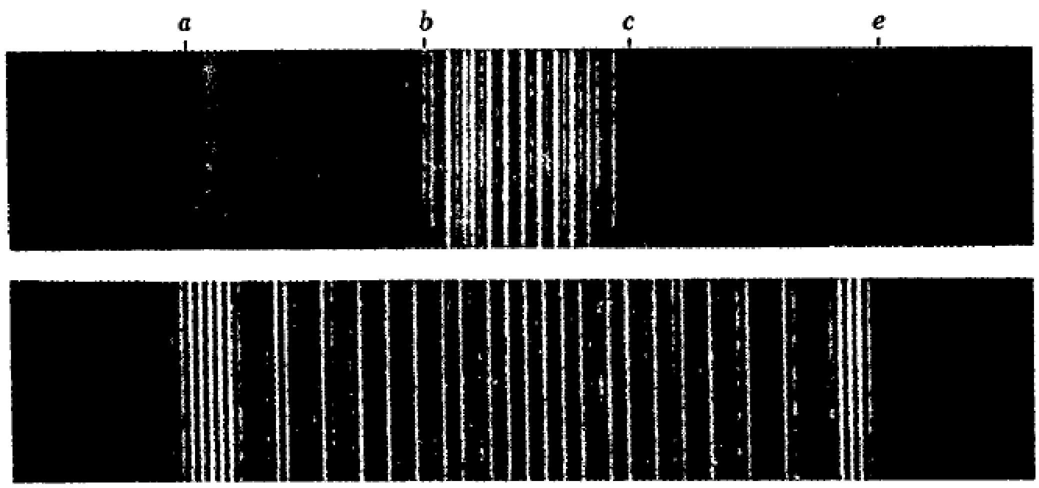

Fig. 5 Interference and diffraction fringes produced in the Fresnel biprism Exp.   .

The wavelength of light can be determined from measurements of the interference fringes produced by the biprism. Calling Band C the distances of the source and screen, respectively, from the prism *P*, *d* the distance between the virtual images S_1 and S_2, and Δx the distance between the successive fringes on the screen, the wavelength of the light is given as

$$\lambda = \frac{\Delta x \, d}{B+C} \text{ --------------------------------- } (8)$$

Thus, the virtual images S_1 and S_2 act like the two slit sources in Young's experiment. In order to find d, the linear separation of the virtual sources, one can measure their angular separation θ on a spectrometer and assume, to sufficient accuracy, that d = B θ. If the parallel light from the collimator covers both halves of the biprism, two images of the slit are produced and the angle θ between these is easily measured with the telescope. At a certain distance from the light the two images can be brought to the point where their inner edges just touch. The diameter of the bulb divided by the distance from the bulb to the prism then gives θ directly. Fresnel biprism are easily made from a small piece of glass, such as half a microscope slide, by beveling about $\frac{1}{8}\ to\ \frac{1}{4}$ in. on one side. This requires very little grinding with ordinary rough materials and polishing with rouge, since the angle required is only about 1^0.

3. Other Apparatus Depending on Division of The Wave Front

Two beams can be brought together in other ways to produce interference. In the arrangement known as *Fresnel's mirrors*, light from a slit is reflected in two plane mirrors slightly inclined to each other. The mirrors produce two virtual images of the slit, as shown in Fig. 6. They act in every respect like the images formed by the biprism, and interference fringes are observed in the region *bc*, where the reflected beams overlap. The symbols in this diagram correspond to those in Fig. 4, and Eq. (8) is again applicable. It will be noted that the angle 2θ subtended at the point of intersection M by the two sources is twice the angle between the mirrors.

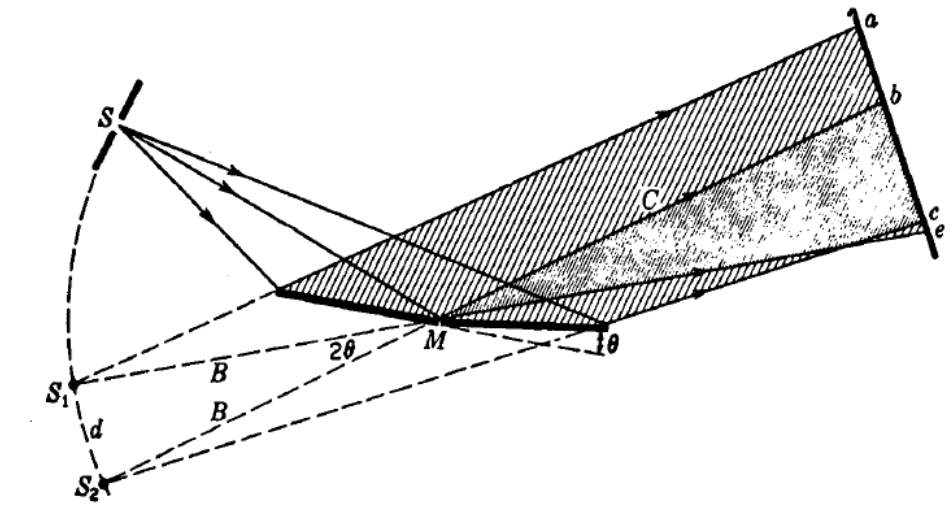

Fig. 6 Geometry of Fresnel mirrors.

An even simpler device, shown in Fig. 7, produces interference between the light reflected in one long mirror and the light coming directly from the source without reflection. In this arrangement, known as Lloyd's mirror, the quantitative relations are similar to those in the foregoing cases, with the slit and its virtual image constituting the double source.

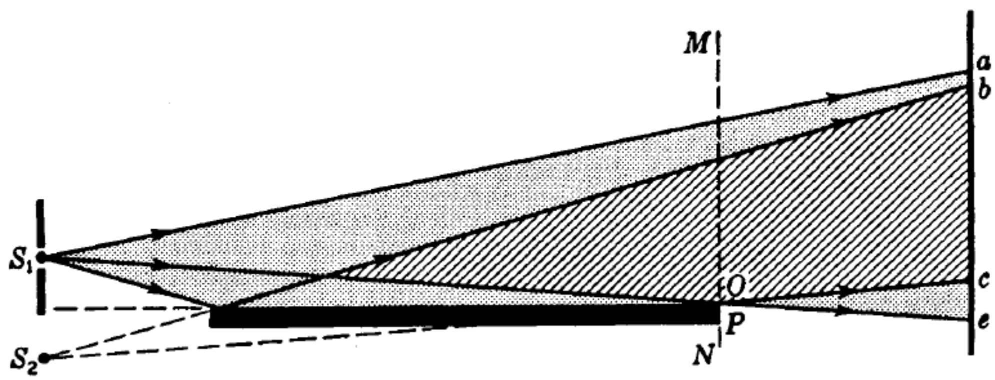

Fig. 7 Geometry of Lloyd's mirror.

An important feature of the Lloyd's-mirror experiment lies in the fact that when the screen is placed in contact with the end of the mirror (in the position *MN*, Fig. 7), the edge *0* of the reflecting surface comes at the center of a *dark* fringe, instead of a bright one as might be expected. This means that one of the two beams has undergone a phase change of π. Since the direct beam could not change phase, this experimental observation is interpreted to mean that the reflected light has changed phase at reflection.

If the light from source S_l in Fig. 7 is allowed to enter the end of the glass plate by. moving the latter up, and to be internally reflected from the upper glass surface, fringes will again be observed in the interval *OP*, with a dark fringe at *O*. This shows that there is again a phase change of π at reflection. In this instance the light is incident at an angle greater than the critical angle for total reflection.

4. Coherent Sources

It will be noticed that the various methods of demonstrating interference so far discussed have one important feature in common: the two interfering beams are always derived from the same source of light. We find by experiment that it is impossible to obtain interference fringes from two separate sources, such as two lamp filaments set side by side. This failure is caused by the fact that the light from anyone source is not an infinite train of waves. On the contrary, there are sudden changes in phase occurring in very short intervals of time (of the order of 10^{-8} s). In Young's experiment and in Fresnel's mirrors and biprism, the two sources S_l and S_2 always have a point-to-point correspondence of phase, since they are both derived from the same source. If the phase of the light from a point in S_l suddenly shifts, that of the light from the corresponding point in S_2 will shift simultaneously. The result is that the *difference* in phase between any pair of points in the two sources always remains constant, and so the interference fringes are stationary. It is a characteristic of any interference experiment with light that the sources must have this point-to-point phase relation, and sources that have this relation are called *coherent sources.*