## 1. MIRRORS

A spherical reflecting surface has image-forming properties similar to those of a thin lens or of a single refracting surface. A spherical mirror is a mirror which has the shape of a piece cut out of a spherical surface. There are two types of spherical mirrors: *concave*, and *convex*. The most commonly occurring examples of concave mirrors are shaving mirrors and makeup mirrors. The most commonly occurring examples of convex mirrors are the passenger-side wing mirrors of cars. These type of mirrors have wider fields of view than equivalent flat mirrors, but objects which appear in them generally look smaller than they actually are.

## 2. FOCAL POINT AND FOCAL LENGTH

Diagrams showing the reflection of a parallel beam of light 1bya concave mirror and by a convex one are given in Fig. 1. A ray striking the mirror at some point such as *T* obeys the law of reflection $\emptyset'' = \emptyset$. All rays are shown as brought to a common focus at *F*,

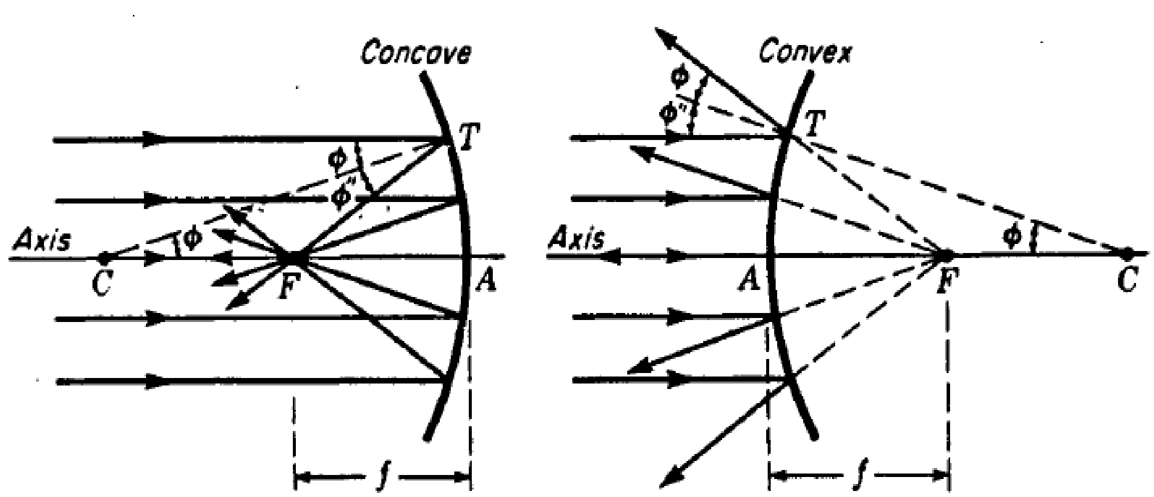


Fig. 1: The primary and secondary focal points of spherical mirrors coincide.

The point $F$ is called the *focal point* and the distance $FA$ the *focal length*. In the second diagram the reflected rays diverge as though they came from a common point $F$. Since the angle $TCA$ also equals $\phi$, the triangle $TCF$ is isosceles, and in general $CF = FT$. But for very small angles $\phi$ (paraxial rays), $FT$ approaches equality with $FA$. Hence

| | |
|---|---|
| $FA = \frac{1}{2} CA \quad , f = -\frac{1}{2} r$ | 1 |

The negative sign is introduced in Eq. (1) so that the focal length of a concave mirror, which behaves like a positive or converging lens, will also be positive. The focal length of a convex mirror, which has a positive radius, will then come out to be negative. This sign convention is chosen as being consistent with that used for lenses; it gives converging properties to a mirror with positive and diverging properties to a mirror with negative $f$. By the principle of reversibility, it can be seen from Fig. 1 that the primary and secondary focal points of a mirror coincide. In other words, it has but one focal point. a transverse plane through the focal point is called the focal plane. Its properties, as shown in Fig. 2, are similar to those of either focal plane of a lens; e.g., parallel rays incident at any angle with the optic axis are brought to a focus at some point in the focal plane. The image $Q'$ of a distant off-axis point object occurs at the intersection with the focal plane of that ray which goes through the center of curvature C.

## 3. MIRROR FORMULAS

In order to be able to apply the standard lens formulas of the preceding chapters to spherical mirrors with as little change as possible, we must adhere to the following sign conventions:

1- *Distances measured from left to right are positive while those measured from right to left are negative.*
2- *Incident rays travel from left to right and reflected rays from right to left.*

3- *The focal length is measured from the focal point to the vertex. This gives f a positive sign for concave mirrors and a negative sign for convex mirrors.*

4- *The radius is measured from the vertex to the center of curvature. This makes r negative for concave mirrors and positive for convex mirrors.*

5- *Object distances s and image distances s' are measured from the object and from the image respectively to the vertex. This makes both sand s' positive and the object and image real when they lie to the left of the vertex; they are negative and virtual when they lie to the right.*

The following is a simple derivation of the formula giving the conjugate relations for a mirror. In Fig. 6G it is observed that by the law of reflection the radius CT bisects the angle MTM'. Using a well-known geometrical theorem, we can then write the proportion

$$\frac{MC}{MT} = \frac{CM'}{M'T}$$

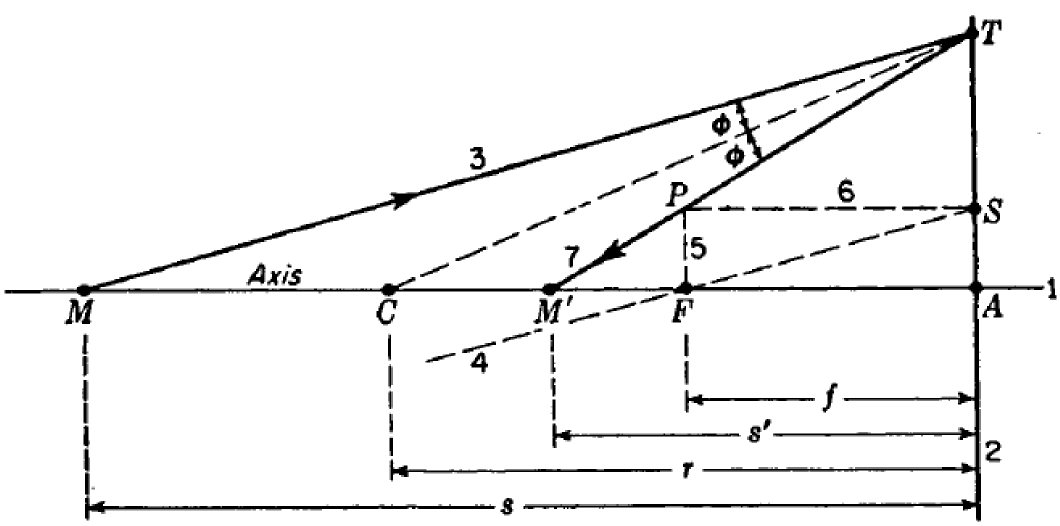


Fig. 2: Oblique-ray method for locating the image formed by a concave mirror.

Now, for paraxial rays, $MT \sim MA = s$ and $M'T \sim M'A = s'$. Also, from the diagram,

$$MC = MA - CA = S + r$$

And $CM' = CA - M'A = -r - S' = -(S' + r)$

Substituting in the above proportion gives

$$\frac{S + r}{S} = -\frac{S' + r}{S'}$$

which can easily be put in the form

| | |
|---|---|
| $\frac{1}{S} + \frac{1}{S'} = -\frac{2}{r}$ | 2 |

**This is mirror formula.**

The primary focal point is defined as that axial object point for which the image is formed at infinity, so substituting $S = f$ and $S' = \infty$ in Eq. (2), we have

$$\frac{1}{f} + \frac{1}{\infty} = -\frac{2}{r}$$

$$f = -\frac{r}{2}$$

The secondary focal point is defined as the image point of an infinitely distant object point. This is $S' = f'$ and $S = \infty$, so that

$$\frac{1}{\infty} + \frac{1}{f'} = -\frac{2}{r}$$

$$f' = -\frac{r}{2}$$

Therefore, the primary and secondary focal points fall together, and the magnitude of the focal length is one-half the radius of curvature. When – *2/r* is replaced by 1/ *f*, Eq. (2) becomes

| | |
|---|---|
| $\frac{1}{S}+\frac{1}{S'}=\frac{1}{f}$ | 3 |

The lateral magnification of the image from a mirror giving by

| | |
|---|---|
| $m=\frac{y'}{y}=-\frac{S'}{S}$ | 4 |

Ex: An object 2.0 cm high is situated 10.0 cm in front of a concave mirror of radius 16.0 cm. Find *(a)* the focal length of the mirror, *(b)* the position of the image, and *(c)* the lateral magnification.

The given quantities are $y$ = +2.0 cm, $s$ = + 10.0 cm, and $r$ =-16.0 cm. The unknown quantities are $f$, $s'$, and $m$.

*(a)* By Eq.

$$f=-\frac{r}{2}=-\frac{-16}{2}=+8\ cm$$

*(b)* By Eq. 3

$$\frac{1}{10}+\frac{1}{S'}=\frac{1}{8}$$

$S' = +40\ cm$

$m = -40/10 = -4$

The image occurs 40.0 cm to the left of the mirror, is 4 times the size of the object, and is real and inverted.

## 4. POWER OF MIRRORS

The power notation that was used in previous lectures to describe the image-forming properties of lenses can readily be extended to spherical mirrors as follows. As definitions, we let

$$P = \frac{1}{f}$$

Ex: An object is located 20.0 cm in front of a convex mirror of radius 50.0 cm. Calculate *(a)* the power of the mirror, *(b)* the position of the image, and *(c)* its magnification.

## 5. THICK MIRRORS

The term *thick mirror* is applied to a lens system in which one of the spherical surfaces is a reflector. Under these circumstances the light passing through the system is reflected by the mirror back through the lens system, from which it emerges finally into the space from which it entered the lens. Three common forms of optical systems that may be classified as thick mirrors are shown in Fig. 3.

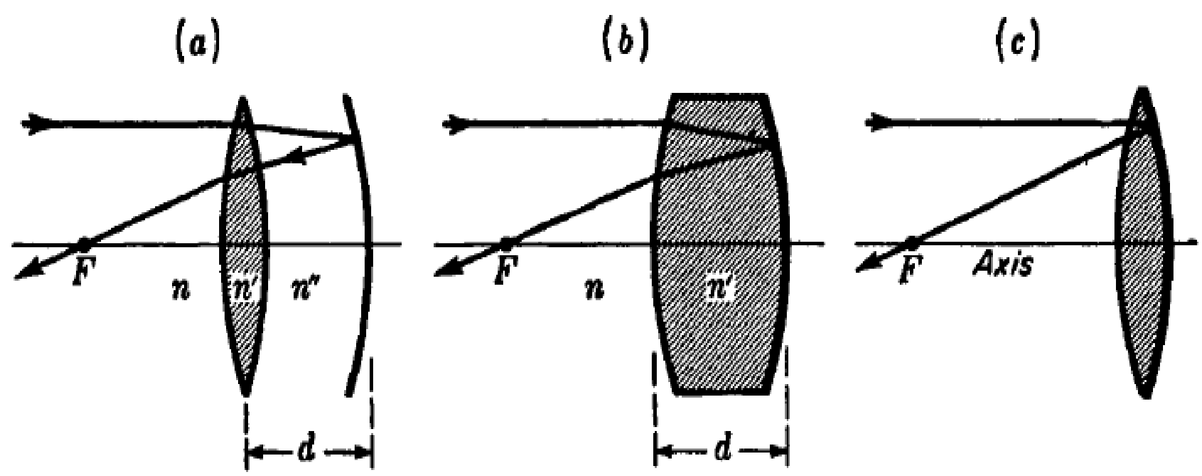


Fig. 3: Diagrams of several types of thick mirrors.

A parallel incident ray is traced through each system to where it crosses the axis, thus locating the focal point. In addition to a focal point and focal plane every thick mirror has a principal point and a principal plane. Two graphical methods by which principal points and planes can be located are given below. The *oblique-ray method* is applied to *(a)* the thin lens and mirror combination in Fig. 4, while the *auxiliary-diagram method* is applied to *(b)* the thick lens and mirror combination in Fig. 5.

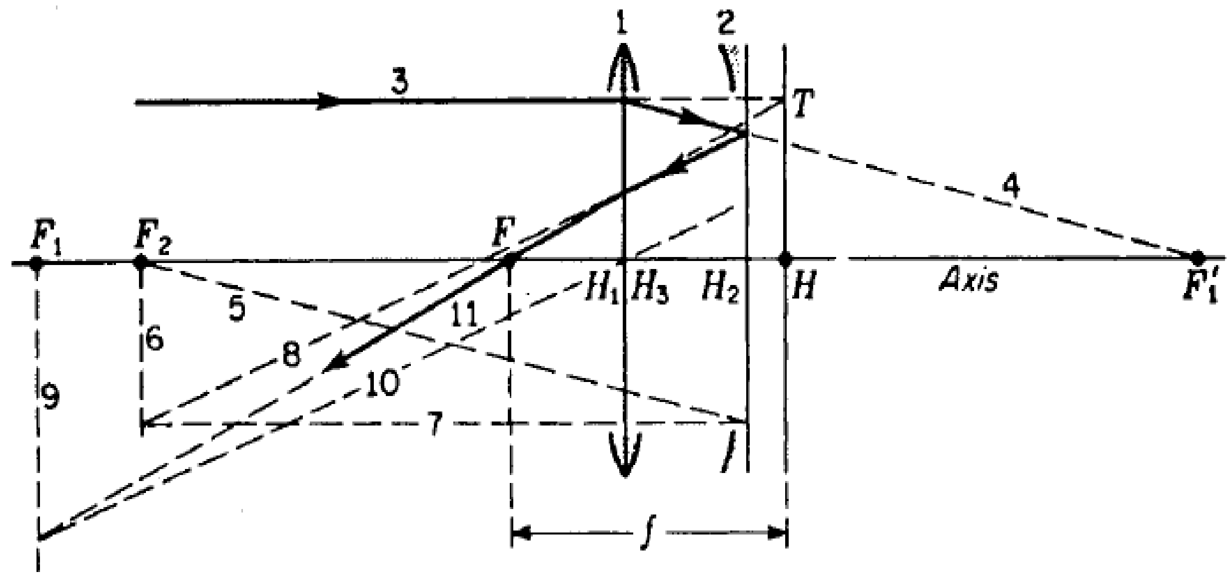


Fig. 4: Oblique-ray construction for locating the focal point and principal point of a thick mirror.

In figure 4, the lens is considered *thin* so that its own principal points may be assumed to coincide at $H_1$, its center. An incident ray parallel to the axis is refracted by the lens, reflected by the mirror, and again refracted by the lens before it crosses the axis of the system at *F*. The point *T* where the incident and final rays, when extended, cross each other locates the principal plane, and *H* represents the principal point. **If** we follow the sign conventions for a single mirror, the focal length *f* of this particular combination is positive and is given by the interval *FH*.

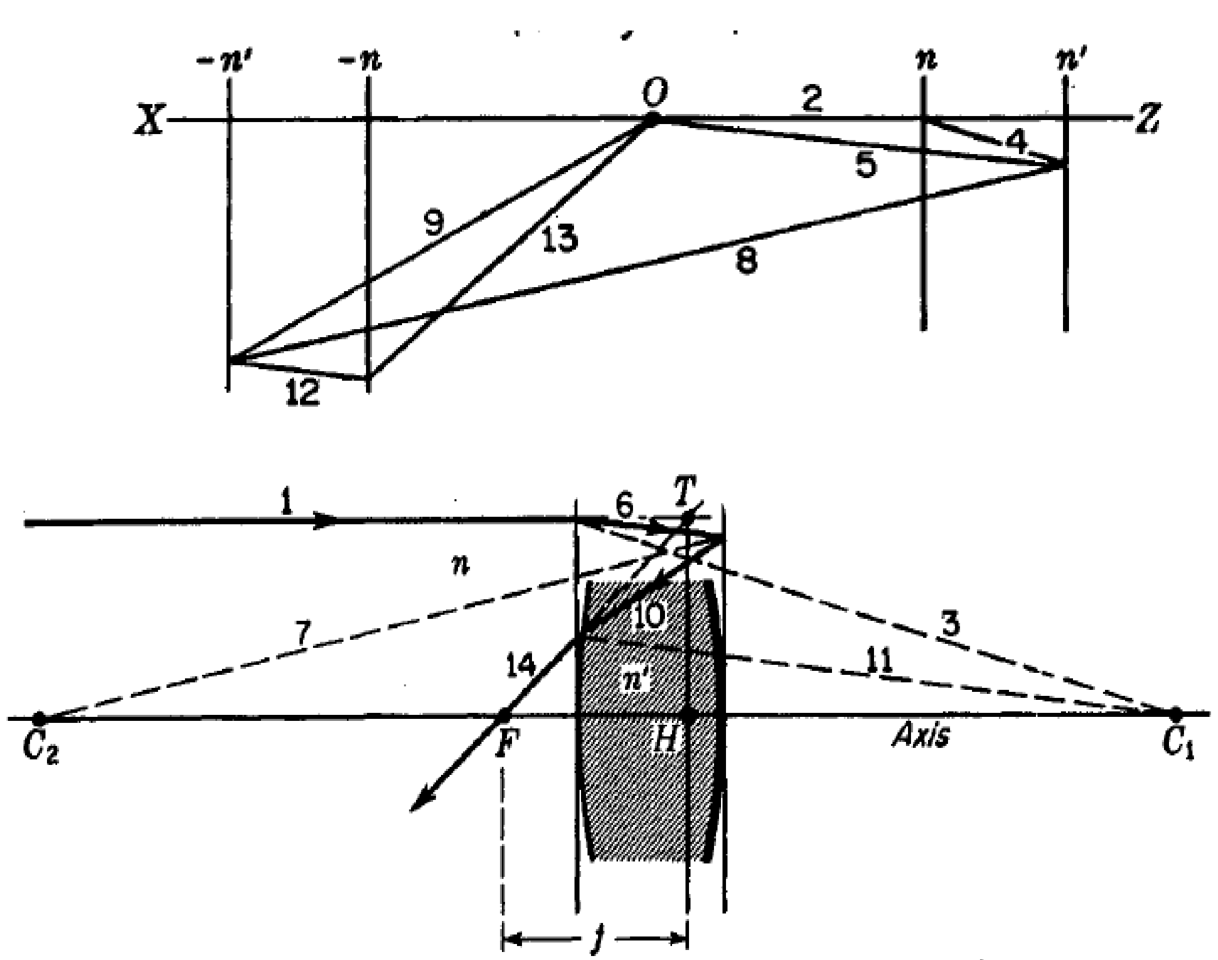


Fig. 5: Auxiliary-diagram method of graphically locating the focal point and principal point of a thick mirror.

**In** Fig. 5, the incident ray is refracted by the first surface, reflected by the second, and finally refracted a second time by the first surface to a point *F* where it crosses the axis. The point *T* where the incident and final rays intersect locates the principal plane and principal point *H*. The graphical ray-tracing construction for this case, shown in the auxiliary diagram in Fig. 5, is started by drawing *XZ* parallel to the axis. With the origin *0* near the center, intervals proportional to n and *n'* are measured off in both directions along *XZ*. After the vertical lines representing n and *n'* are drawn, the remaining lines are drawn in the order of the numbers 1,2,3, .... Each even-numbered line is drawn parallel to its preceding odd-numbered line.

## 6. THICK-MIRROR FORMULAS

These formulas will be given in the power notation for case (a) shown in Fig. 3. When $r_1$, $r_2$, and $r_3$ are the radii of the three surfaces consecutively from left to right, the power of the combination can be shown* to be given by

| | |
|---|---|
| $P = (1 - c\,P_1)(\,2P_1 + \,P_2 - c\,P_1P_2\,)$ | 5 |

where, for the case in diagram *(a)* only and $n'' = n$,

| | |
|---|---|
| $P_1 = (n' - n)(\,K_1 + \,K_2\,)$ | 6 |

| | |
|---|---|
| $P_2 = -2n\,K_3$ | 7 |

$$K_1 = \frac{1}{r_1} \quad , K_2 = \frac{1}{r_2} \quad , \qquad K_3 = \frac{1}{r_3}$$

Of the refractive indices, $n'$ represents that of the lens and $n$ that of the surrounding space. The distance from the lens to the principal point of the combination is given by

| | |
|---|---|
| $H_1H = \dfrac{c}{1 - c\,P_1}$ | 8 |

where $H_I$ is located at the center of the lens and

c = d / n

## 7. SPHERICAL ABERRATION

The discussion of a single spherical mirror in the preceding sections has been confined to paraxial rays. Within this rather narrow limitation, sharp images of objects at any distance may be formed on a screen. If, however, the light is not confined to the paraxial region, all rays from one object point do not come to a focus at a common point and we have an undesirable effect known as *spherical aberration.* The phenomenon is illustrated in Fig. 6. where parallel incident rays at increasing distances *h* cross the axis closer to the mirror. The envelope of all rays forms what is known as a *caustic surface.* If a small screen is placed at the paraxial focal plane F and then moved toward the mirror, a point is reached where the size of the circular image spot is a minimum. This disk like spot is indicated in the diagram and is called the *circle of least confusion.*

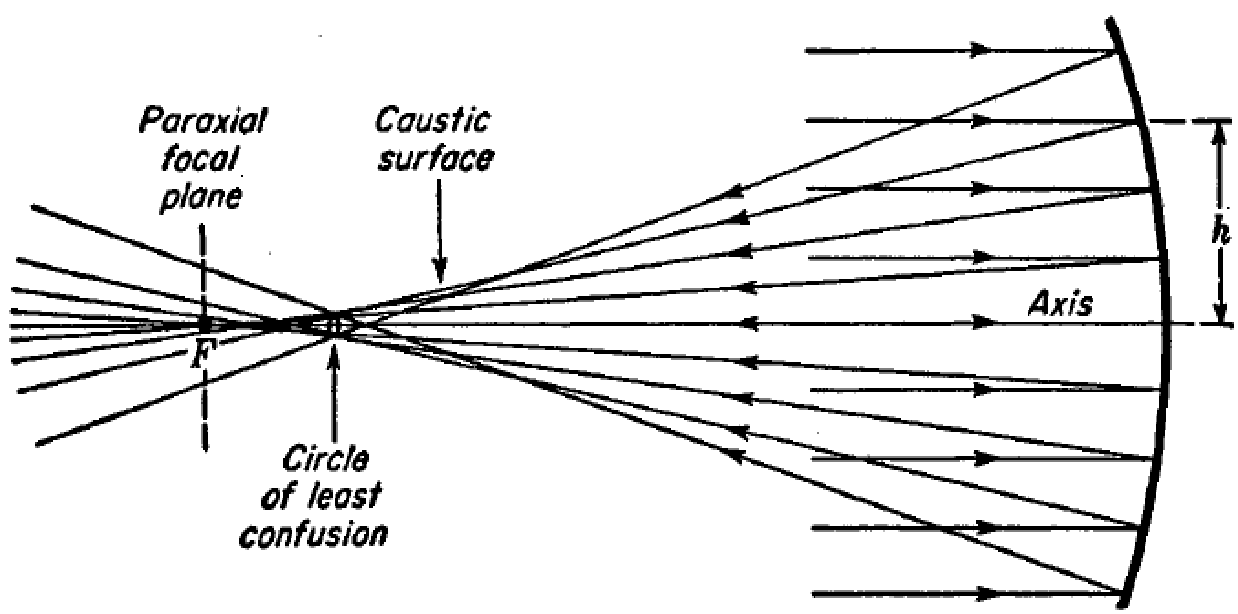


Fig. 6: Spherical aberration of a concave spherical mirror.

Over the past years numerous methods of reducing spherical aberration have been devised. If instead of a spherical surface the mirror form is that of a paraboloid of revolution, rays parallel to the axis are all brought to a focus at the same point as in Fig. 7(a). Another method is known as a *Mangin mirror*, is shown in Fig. 7(b). Here a meniscus lens is employed in which both surfaces are spherical. When the back surface is silvered to form the concave mirror, all parallel rays are brought to a reasonably good focus.

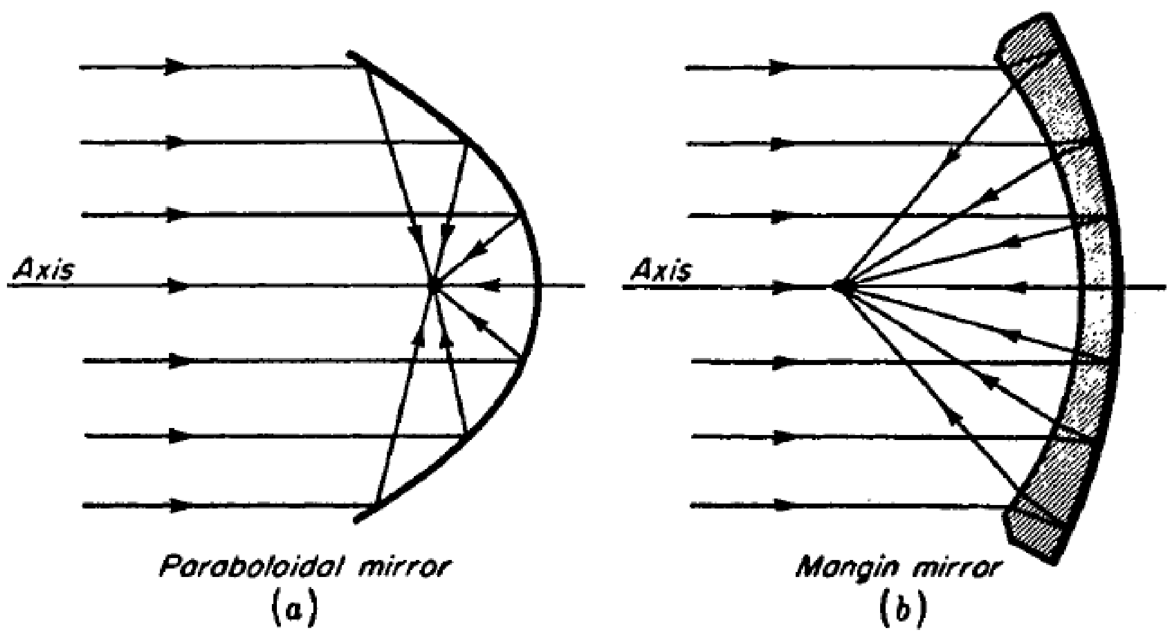


Fig. 7: *(a)* Concave parabolic mirror and *(b)* concave spherical mirror, corrected for spherical aberration.

## 8. ASTIGMATISM

This defect of the image occurs when an object point lies some distance from the axis of a concave or convex mirror. The incident rays, whether parallel or not, make an appreciable angle ϕ with the mirror axis. The result is that, instead of a point image, two mutually perpendicular line images are formed. This effect is known as *astigmatism* and is illustrated by a perspective diagram in Fig. 8. Here the incoming rays are parallel while the reflected rays are converging toward two lines Sand *T*. The reflected rays in the vertical or *tangential* plane ***RASE*** are seen to cross or to focus at *T*, while the fan of rays in the horizontal or *sagittal* plane ***JAKE*** cross or focus at S. If a screen is placed at *E* and moved toward the mirror, the image will become a vertical line at S, a circular disk at *L*, and a horizontal line at *T*.

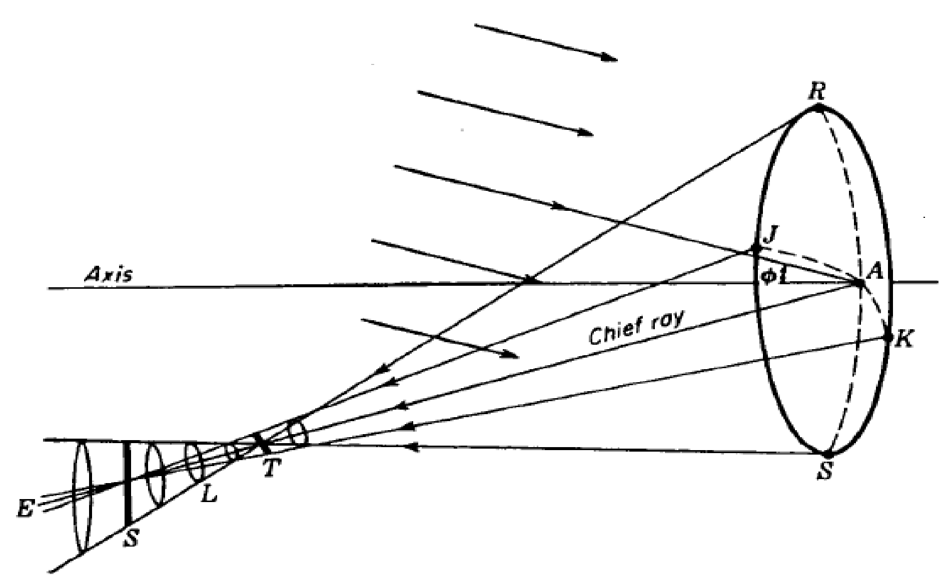


Fig. 8: Astigmatic images of an off-axis object point at infinity, as formed by a concave spherical mirror. The lines *T* and *S* are perpendicular to each other.