## 1. LATERAL MAGNIFICATION

A simple formula for the image magnification produced by a single lens can be derived from the geometry of Fig. 1. By construction it is seen that the right triangles $QMA$ and $Q'M'A$ are similar. Corresponding sides are therefore proportional to each other, so that

$$\frac{M'Q'}{MQ} = \frac{AM'}{AM} \tag{1}$$

where $AM'$ is the image distance $s'$ and $AM$ is the object distance $s$. Taking upward directions as positive, $y = MQ$, and $y' = -M'Q'$; so we have by direct substitution $y'/y = -s'/s$. The lateral magnification is therefore

$$m = \frac{y'}{y} = -\frac{s'}{s} \tag{2}$$

When s and $s'$ are both positive, as in Fig. 1, the negative sign of the magnification signifies an inverted image.
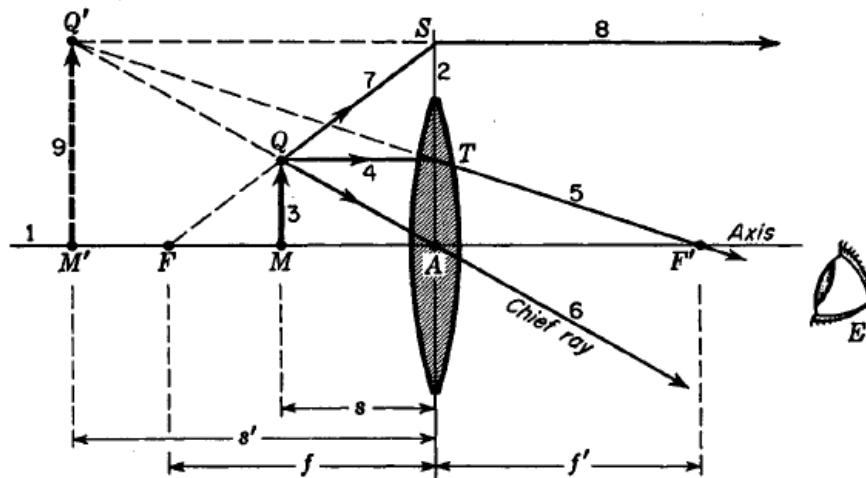


Fig.1
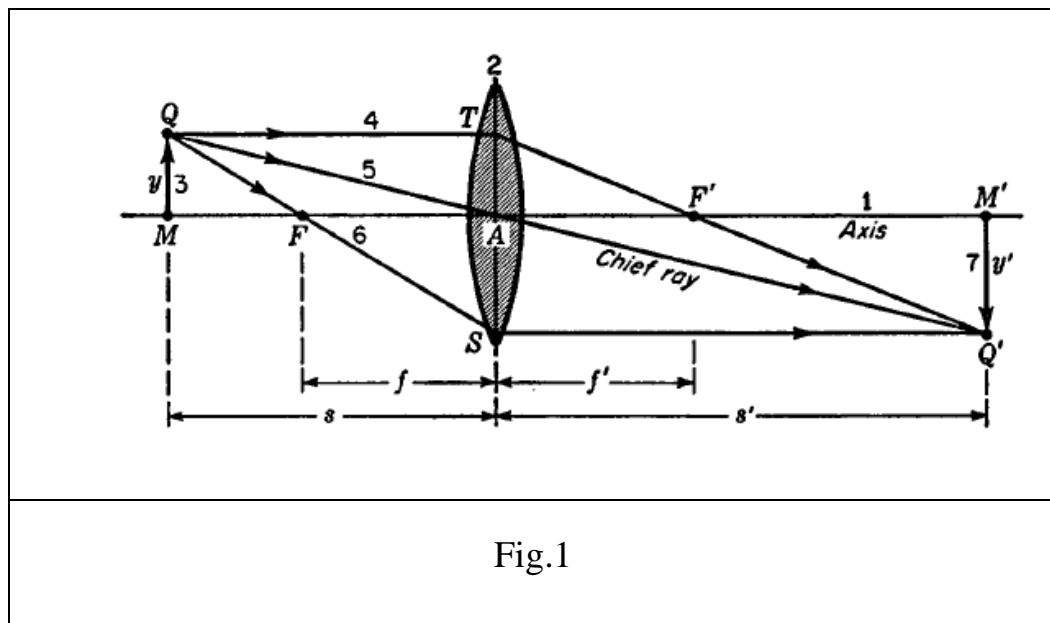
Ex: If an object is located 6.0 cm in front of a lens of focal length + 10.0 cm, where will the image be formed?

$$s' = \frac{s \times f}{s - f} \quad , s' = \frac{(+6) \times (+10)}{(+6) - (+10)} = \frac{+60}{-4} = -15$$

The minus sign indicates that the image lies to the left of the lens. Such an image is always virtual. The magnification is obtained by

$$m = -\frac{s'}{s} = -\frac{-15}{+6} = +2.5$$

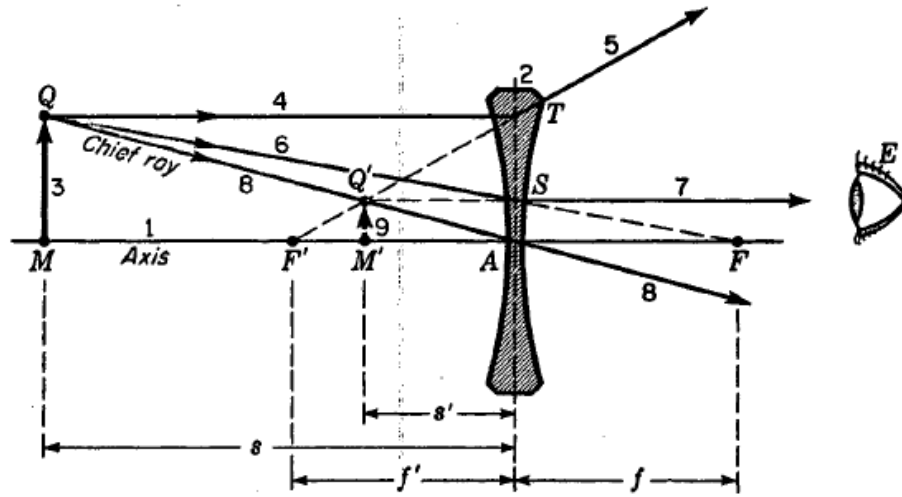The positive sign means that the image is erect.



Ex: An object is placed 12.0 cm in front of a diverging lens of focal length 6.0 cm. Find the image.

The given quantities are $s = + 12.0$em and! = -6.0 cm, while the unknown quantities are $s'$ and $m$.

$$s' = \frac{(+12) \times (-6)}{(+12) - (-6)} = \frac{-72}{+18} = -4 \; cm$$

$$m = -\frac{s'}{s} = -\frac{-4}{+12} = +\frac{1}{3}$$



The image is therefore to the left of the lens, virtual, erect, and one-third the size of the object.

## 2.  LENS MAKERS' FORMULA

If a lens is to be ground to some specified focal length, the refractive index of the glass must be known. It is customary for manufacturers of optical glass to specify the refractive index for yellow sodium light, the D line. Supposing the index to be known, the radii of curvature must be so chosen as to satisfy the equation

$$\frac{1}{f} = (n-1)(\frac{1}{r_1} - \frac{1}{r_2})$$

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As the rays travel from left to right through a lens, *all convex surfaces are taken as having a positive radius and all concave surfaces a negative radius.* For an equiconvex lens $r_1$ for the first surface is positive and $r_2$ for the second surface negative. Substituting the value of $1/f$ , we write

$$\frac{1}{s} + \frac{1}{s'} = (n-1)(\frac{1}{r_1} - \frac{1}{r_2}) \qquad\qquad 4$$

Ex: plano-convex lens having a' focal length of 25.0 cm is to be made of glass of refractive index $n = 1.520$. Calculate the radius of curvature?

Since a plano-convex lens has one flat surface, the radius for that surface is infinite, and r1 in Eq. (3) is replaced by 00. The radius $r2$ of the second surface is the unknown.

$$\frac{1}{25} = (1.520 - 1)(\frac{1}{\infty} - \frac{1}{r_2})$$

$$\frac{1}{25} = (0.520)\left(0 - \frac{1}{r_2}\right) = -\frac{0.520}{r_2}$$

$r_2 = - (25 \times 0.520) = - 13$ cm

If this lens is turned around, as in the figure, we shall have r1 $= + 13.0$ cm and $r2 = 00$.

## 3. THE POWER OF A THIN LENS

The power of a thin lens in diopters is given by the reciprocal of the focal length in meters:

$$P = \frac{1}{f} \qquad diopters = \frac{1}{focal\ length, m}$$

For example, a lens with a focal length of $+ 50.0$ cm has a power of 1/0.50 m $= + 2$ D *(P $= + 2.0$ D)*, whereas one of - 20.0 cm focal length has a power of 1/0.20m= - 5 D *(P $= -$ 5.0 D)*, etc. Converging lenses have a positive power, while diverging

lenses have a negative power. By making use of the lens makers' formula [Eq. (3)] we can write

$$P = (n - 1)(\frac{1}{r_1} - \frac{1}{r_2})$$

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where $r_1$ and $r_2$ are the two radii, **measured in meters**, and $n$ is the refractive index of the glass.

Ex: The radii of both surfaces of an equiconvex lens of index 1.60 are equal to 8.0 cm. Find its power.

The given quantities to be used in Eq. (5) are $n = 1.60$, $r_1 = 0.080$ m, and $r_2 = -0.080$ m

$$P = (n - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = (1.6 - 1)\left(\frac{1}{0.08} - \frac{1}{-0.08}\right) = 0.6 \times \frac{2}{0.08} = +15\ D$$

## 4.  THIN LENSES IN CONTACT

When two thin lenses are placed in contact, as shown in Fig. 2, the combination will act as a single lens with two focal points symmetrically located at $F$ and $F'$ on opposite sides. Parallel incoming rays are shown refracted by the first lens toward its secondary focal point $F'_1$. Further refraction by the second lens brings the rays together at $F'$. This latter is defined as the secondary focal point of the combination, and its distance from the center is defined as the combination's secondary focal length $f'$.  If we now apply the simple lens formula to the rays as they enter and leave the second lens $L_2$, we note that for the second lens alone $f'_1$ is the object distance (taken with a negative sign), $f'$ is the image distance, and $f'$ is the focal length. When lens Eq. is applied, these substitutions for $s$, $s'$, and $f$, respectively, give
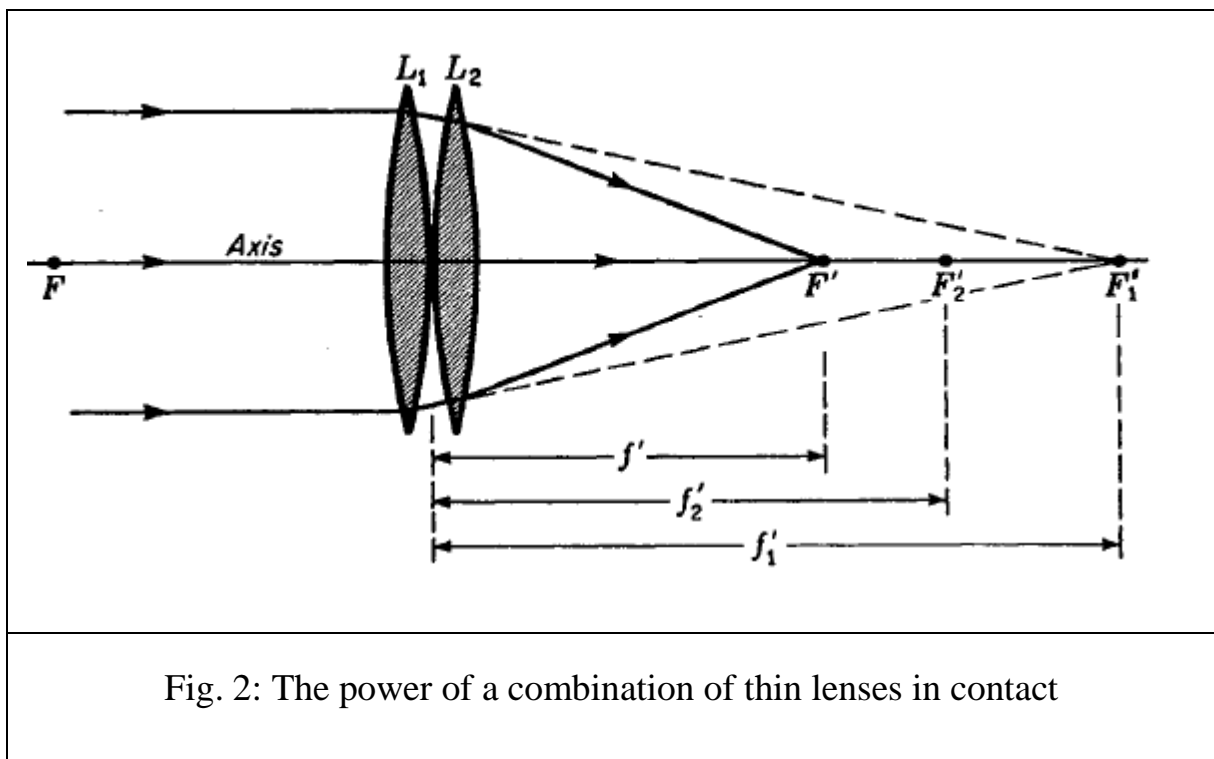
$$\frac{-1}{f_1'} + \frac{1}{f'} = \frac{1}{f_2'} \quad or \quad \frac{1}{f'} = \frac{1}{f_1'} + \frac{1}{f_2'}$$

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Since we have assumed that the lenses are in air, the primary focal lengths are all equal to their respective secondary focal lengths and we can drop all primes and write

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

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we can write $P_1 = 1/f_1$ , $P_2 = 1/f_2$, and $P = 1/f$ we obtain for the power of the combination

$P = P_1 + P_2$



Fig. 2: The power of a combination of thin lenses in contact

## 5. DERIVATION OF THE LENS MAKERS' FORMULA

The geometry required for this derivation is shown in Fig. 3. Let $n$, $n'$, and $n''$ represent the refractive indices of the three media as shown , $f_1$ and $f'_1$ the focal lengths for the first surface alone, and $f'_2$ and $f''_2$ the focal lengths for the second surface alone.
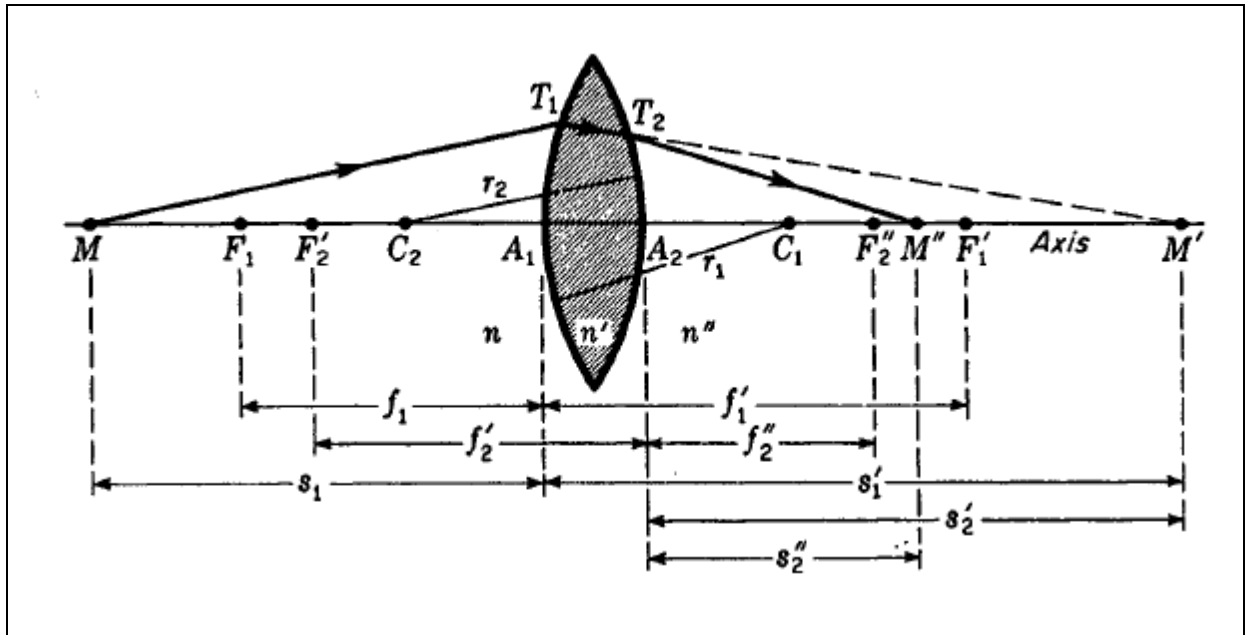


Fig. 3: Each surface of a thin lens has its own focal points and focal lengths, as well as separate object and image distances.

The oblique ray $MT_1$ is incident on the first surface as though it came from an axial object point $M$ at a distance $S_1$ from the vertex $A_1$. At $T_1$ the ray is refracted according to Eq. (#) and is directed toward the conjugate point $M'$:

| | |
|---|---|
| $$\frac{n}{S_1} + \frac{n'}{S'_1} = \frac{n'-n}{r_1}$$ | 8 |

$$\frac{n}{s} + \frac{n'}{s'} = \frac{n'-n}{r} \quad \#$$

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Arriving at $T_2$, the same ray is refracted in the new direction $T_2M''$. For this second surface the object ray $T_1 T_2$ has for its object distance $S_2$, and the refracted ray gives an image distance of $s_2'$ When Eq. (#) is applied to this second refracting surface,

$$\frac{n'}{s_2'} + \frac{n''}{s_2''} = \frac{n'' - n'}{r_2}$$

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If we now assume the lens thickness to be negligibly small compared with the object and image distances, we note the image distance $s_1'$ for the first surface becomes equal in magnitude to the object distance $s_2'$ for the second surface. Since $M'$ is a virtual object for the second surface, the sign of the object distance for this surface is negative. we can set $s_1' = -s_2'$ and write

$$\frac{n'}{s_1'} = -\frac{n'}{s_2'}$$

If we now add Eqs. (8) and (9) and substitute this equality, we obtain

$$\frac{n}{s_1} + \frac{n''}{s_2''} = \frac{n' - n}{r_1} + \frac{n'' - n'}{r_2}$$

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If we now call $S_1$ the object distance and designate it s as in Fig. 4 and call $s_2''$ the image distance and designate it S", we can write Eq. (10) as

$$\frac{n}{s} + \frac{n''}{s''} = \frac{n' - n}{r_1} + \frac{n'' - n'}{r_2}$$

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$$\frac{n}{f} = \frac{n' - n}{r_1} + \frac{n'' - n'}{r_2} = \frac{n''}{f''}$$

12

8

$$\frac{f}{f''} = \frac{n}{n''}$$

If the medium on both sides of the lens is the same, $n = n''$,

| | |
|---|---|
| $\dfrac{n}{s} + \dfrac{n''}{s''} = (n' - n)(\dfrac{1}{r_1} - \dfrac{1}{r_2})$ | 12 |

if the surrounding medium is air $(n = 1)$, we obtain the lens makers' formula

$$\frac{1}{s} + \frac{1}{s''} = (n' - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$



Fig. 4: When the media on the two sides of a thin lens have different indices, the primary and secondary focal lengths are not equal and the ray through the lens center is deviated.