Lecture (8)

Electricity and Magnetism II

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A Charged capacitor connected to an inductor

The sign convention for an inductor has already been stated in the previous section. A capacitor behaves like a resistor when losing charge (or losing electrical energy) and behaves like a source when it is being charged (that is gaining electrical energy). So both resistor and source sign conventions apply depending in the situation. In both cases, if the capacitor is transverse in the direction of the current, the potential difference is given as $-\frac{Q}{C}$. Now, applying Kirchhoff's loop rule,

$$-\frac{Q}{C} - L\frac{dI}{dt} = 0 \quad with \quad I = \frac{dQ}{dt}$$

After some arrangement, the charge stored in the capacitor satisfies the differential equation.

$$\frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0$$

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The general solution of this differential equation is:

$$\boldsymbol{Q = C_1 \cos\left(\sqrt{\frac{1}{LC}}\,t + C_2\right)}$$

Where $\boldsymbol{C_1}$ and $\boldsymbol{C_2}$ are arbitrary constant to be determined from two initial (or other) conditions. Let the initial charge of the capacitor be $\boldsymbol{Q_{max}}$. Then,

$$\boldsymbol{Q(t = 0) = C_1 \cos\left(\sqrt{\frac{1}{LC}}\,(0) + C_2\right) = Q_{max}}$$

$$\boldsymbol{Q_{max} = C_1 \cos(C_2)}$$

The current is given by:

$$\boldsymbol{I = \frac{dQ}{dt} = -\frac{C_1}{\sqrt{LC}}\sin\left(\sqrt{\frac{1}{LC}}t + C_2\right)}$$

The initial current is zero, Therefore,

$$\boldsymbol{I(t=0) = -\frac{C_1}{\sqrt{LC}}\sin\left(\sqrt{\frac{1}{LC}}(0) + C_2\right) = 0}$$

$$\boldsymbol{0 = -\frac{C_1}{\sqrt{LC}}\sin(C_2)}$$

Which implies $\boldsymbol{C_2 = 0}$ (the possibility $\boldsymbol{C_1 = 0}$ is avoided because it implies zero charge).

With $C_2 = 0$, it follows that $C_1 = Q_{max}$ and the solutions for the charge and current as a function of time are respectively given as

$$Q(t) = Q_{max}\cos\left(\frac{1}{\sqrt{LC}}t\right) \qquad and \qquad I(t) = -\frac{Q_{max}}{\sqrt{LC}}\sin\left(\frac{1}{\sqrt{LC}}t\right)$$

The coefficient of time $\frac{1}{\sqrt{LC}}$ represents the angular frequency (number of radians executed per second) and is denoted by ω_o.

$$\omega_o = \frac{1}{\sqrt{LC}}$$

The frequency f_o (Number of cycles per second) is related with angular frequency by:

$$\omega_o = 2\pi f_o \quad \Longrightarrow \quad f_o = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

The period (time taken for one cycle) is the inverse of the frequency, $\quad T_o = 2\pi\sqrt{LC}$

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Energy of an LC Circuit

From Kirchhoff's loop rule $\mathbf{L}\frac{\boldsymbol{dI}}{\boldsymbol{dt}} = -\frac{\boldsymbol{Q}}{\boldsymbol{C}}$, multiplying both sides by $\boldsymbol{dQ}$,

$$\boldsymbol{L\,dI\,\frac{dQ}{dt} = -\frac{Q}{C}dQ}$$

Replacing $\frac{\boldsymbol{dQ}}{\boldsymbol{dt}}$ by **I** results in $\boldsymbol{L\,I\,dI = -\frac{Q}{C}dQ}$. Integrating this equation gives

$$\boldsymbol{\frac{1}{2}LI^2 + \frac{1}{2}\frac{Q^2}{C} = c}$$

Where $\boldsymbol{c}$ stand for an integration constant. But the expressions $\boldsymbol{\frac{1}{2}LI^2}$ and $\boldsymbol{\frac{1}{2}\frac{Q^2}{C}}$ represent the magnetic energy stored in the inductor and the electrical energy stored in the capacitor respectively.

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Therefore, it follows that the sum of the electrical energy in the capacitor and the magnetic energy stored in the inductor is a constant independent of time and should be equal to the initial electrical energy in the capacitor $\frac{Q_{max}^2}{2\,C}$ that is,

$$U_{em} = \frac{1}{2} L I^2 + \frac{1}{2} \frac{Q^2}{C} = \frac{Q_{max}^2}{2\,C}$$

Where U_{em} stands for the total energy stored in the **LC** circuit. The process involves only the interchange between electrical and magnetic energy.

Example 1: a 20 F capacitor is connected to a 10 V battery. Then it is disconnected from the battery and connected to a 5 H inductor. (a) calculate the maximum charge of the capacitor (b) calculate the angular frequency of the circuit (c) how many cycles does the circuit execute per second (d) calculate the time taken for one cycle (e) give expressions for the charge and current as a function of time

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Example 2: A 16 F capacitor is connected to an 8 V battery and disconnected from the battery. Then it is connected to a 4 H inductor. (a) calculate the total energy of the circuit at any time (2) calculate the current in the circuit when the charge is half of the maximum value (3) Calculate the maximum value of the current

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Transformer

A transformer is a device used to increase or decrease the amplitude of an ac voltage (An ac voltage is time varying voltage that varies with time typically like sine or cosine). It consists of two coils wound on a magnetic material such as iron. One of the coils called the primary coil is connected to an ac source. The second coil which gives the output is called the secondary coil. The current in the primary coil produces magnetic field in the magnetic material uniformly. Thus, this field crosses both coils. Since the current is changing with time (and hence the magnetic field), according to Faraday's law, there will be induced emf in both coils given by:

$$\boldsymbol{\varepsilon_1 = -N_1 \frac{d\phi_1}{dt} \qquad and \qquad \varepsilon_2 = -N_2 \frac{d\phi_2}{dt}}$$

Where $\boldsymbol{N_1}$ and $\boldsymbol{N_2}$ are number of turns in the primary and secondary coils respectively; and $\boldsymbol{\varepsilon_1}$ and $\boldsymbol{\varepsilon_2}$ are induced emf in coil 1 and coil 2 respectively.

From Kirchhoff's rule $\boldsymbol{\varepsilon_1}$ is equal to the input voltage in the primary. $\boldsymbol{\phi_1}$ and $\boldsymbol{\phi_2}$ are fluxes crossing coil1 and coil 2 respectively.

But since both coils are crossed by the same magnetic field and the cross sectional area of the magnetic material is the same for both, $\boldsymbol{\phi_1}$ and $\boldsymbol{\phi_2}$ are equal ($\boldsymbol{\phi_1 = \phi_2 = BA}$). Hence, the emf may be written as,

$$\frac{\varepsilon_1}{N_1} = -\frac{d\phi_1}{dt} \qquad and \qquad \frac{\varepsilon_2}{N_2} = -\frac{d\phi_1}{dt}$$

Which implies, $$\frac{\varepsilon_1}{N_1} = \frac{\varepsilon_2}{N_2} \quad or \quad \varepsilon_2 = \frac{N_2}{N_1}\,\varepsilon_1$$

This implies that the output voltage can be adjusted to a desired voltage by the coil of a suitable $\frac{N_2}{N_1}$ ratio. A transformer whose effect is to increase voltag ... $N_2 > N_1$ is called a step up transformer; and a transformer whose effect is to decre ... called a step down transformer.

Example: The primary coil and the secondary coil of a transformer have 50 and 125 turns respectively. The primary coil is connected to an ac voltage of amplitude 10 V. (a) Calculate the amplitude of the voltage obtained from the secondary coil. (b) Is this a step up or step down transformer?

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