Lecture (5)

Electricity and Magnetism II

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Magnetic Flux

The magnetic flux across any surface, closed or not, placed in a magnetic field is:

$$\emptyset_B = \int_S \vec{B} . \vec{u}_N \, ds$$

The magnetic flux, being magnetic field times area, is expressed in $T\, m^2$, a unit called **Weber**. It is abbreviated **wb**, so that

$$wb = T\, m^2 = m^2\, kg\, s^{-1}\, C^{-1}$$

Since,

$$T = kg\, s^{-1}\, C^{-1}$$

Since there are no masses or poles, the lines of the magnetic field $\vec{B}$ are closed. Therefore, if we consider a closed surface in a magnetic field, the inward magnetic flux is equal to the outward magnetic flux. We then conclude that, The flux of the magnetic field through a closed surface is always zero.

Therefore, we may write: $\oint_S \vec{B} . \vec{u}_N \, ds = 0$

Gauss' Law for Magnetism

The net magnetic flux out of any closed surface is zero. This amounts to a statement about the sources of magnetic field. For a magnetic dipole, any closed surface the magnetic flux directed inward toward the south pole will equal the flux outward from the north pole. The net flux will always be zero for dipole sources. If there were a magnetic monopole source, this would give a non-zero area integral. The divergence of a vector field is proportional to the point source density, so the form of Gauss' law for magnetic fields is then a statement that there are no magnetic monopoles.

Integral form

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Differential form

$$\nabla \cdot B = 0$$

The Magnetization Vector

The magnetization gives rise to a net current $\boldsymbol{I_{mag}}$ on the surface of the material, to behave as a solenoid.

The magnetization Vector (**M**) of material is defined as the magnetic moment of a medium per unit volume. If (**m**) is the magnetic dipole for each atom, and (**n**) is the number of atoms per unit volume, the magnetization is:

$$\vec{\boldsymbol{M}} = \boldsymbol{n}\,\vec{\boldsymbol{m}}$$

The magnetization $\vec{\boldsymbol{M}}$ is expressed in $\boldsymbol{A\,m^{-1}\ or\ m^{-1}\,s^{-1}\,C}$ units, and the equivalent to the current per unit length.

$$\vec{\boldsymbol{M}} = \boldsymbol{I_{mag}}$$

The Magnetization Field (H)

Let us consider a cylindrical piece of matter placed inside a long solenoid which is carrying a current **I**. this current produces a magnetic field that magnetizes the cylinder and gives rise to magnetization surface current on the cylinder in the same direction as (**I**).

If a solenoid has (**n**) turns per unit length, the system of solenoid carrying a current per unit length equal to $(\boldsymbol{n\,I + I_{mag}})$ or $(\boldsymbol{n\,I + M})$.

This effective solenoid current gives rise to a resultant magnetic field (**B**) parallel to the axis of the cylinder.

$$\boldsymbol{B = \mu_o(n\ \ I + M)} \quad \boldsymbol{or} \quad \boldsymbol{\left(\frac{1}{\mu_o}\,B - M\right) = n\,I}$$

($\boldsymbol{n\,I}$) is the free currents per unit length on the surface of cylinder, and is called the magnetizing field (**H**):

$$\vec{H} = \frac{1}{\mu_o}\vec{B} - \vec{M}$$

$$B = \mu_o(H + M)$$

Unit of (**H**) is **A/m**.

Consider (**L**) is the length along the surface, then the total free current on the cylinder surface is:

$$I_{free} = H\,L$$

If $\boldsymbol{\Lambda_H}$ is the circulation of (**H**) around the length **L**.

$$\Lambda_H = \oint_L \vec{H}.\vec{dL} = H\,L = I_{free}$$

Magnetic Susceptibility (χ_m) and Permeability (μ)

$$\vec{B} = \mu_o(\vec{H} + \vec{M})$$

The relation between **M** and **H** is

$$\vec{M} = \chi_m \vec{H}$$

Where χ_m is called the magnetic susceptibility of the material and is expresses the response of a medium to an external magnetic field, therefore,

$$\vec{B} = \mu_o(\vec{H} + \chi_m \vec{H}) = \mu_o(1 + \chi_m)\vec{H} = \mu \vec{H}$$

Where,

$$\mu = \frac{B}{H} = \mu_o(1 + \chi_m) \qquad and \qquad \mu_r = \frac{\mu}{\mu_o} = 1 + \chi_m$$

are the magnetic permeability and relative permeability of matter.

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Magnetic Field Strength H

The magnetic fields generated by currents and calculated from Ampere's Law or the Biot-Savart Law are characterized by the magnetic field **B** measured in Tesla. But when the generated fields pass through magnetic materials which themselves contribute internal magnetic fields, ambiguities can arise about what part of the field comes from the external currents and what comes from the material itself. It has been common practice to define another magnetic field quantity, usually called the "magnetic field strength" designated by **H**. It can be defined by the relationship

$$H = B/\mu_m = B/\mu_0 - M$$

and has the value of unambiguously designating the driving magnetic influence from external currents in a material, independent of the material's magnetic response.

The relationship for **B** can be written in the equivalent form

$$B = \mu_0(H + M)$$

H and **M** will have the same units, **amperes/meter**. To further distinguish **B** from **H, B** is sometimes called the magnetic flux density or the magnetic induction. The quantity **M** in these relationships is called the magnetization of the material.

Another commonly used form for the relationship between **B** and **H** is

$$B = \mu_m H$$

where

$$\mu = \mu_m = \mu_r\, \mu_0$$

μ_0 being the magnetic permeability of space and μ_r the relative permeability of the material.

If the material does not respond to the external magnetic field by producing any magnetization, then $\mu_r = 1$. Another commonly used magnetic quantity is the **magnetic susceptibility which specifies how much the relative permeability differs from one**.

$$\textbf{Magnetic susceptibility } \chi_m = \mu_r - 1$$

For paramagnetic and diamagnetic materials the relative permeability is very close to 1 and the magnetic susceptibility very close to zero. For ferromagnetic materials, these quantities may be very large.

The unit for the magnetic field strength **H** can be derived from its relationship to the magnetic field **B, B=μH**. Since the unit of magnetic permeability **μ** is **N/A²**, then the unit for the magnetic field strength is:

$$T/(N/A^2) = (N/Am)/(N/A^2) = A/m$$

An older unit for magnetic field strength is the **oersted**:

1 A/m = 0.01257 **oersted**.

Maxwell's Equations

1. Gauss' law for electricity

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0} = 4\pi k\rho$$

2. Gauss' law for magnetism

$$\nabla \cdot B = 0$$

3. Faraday's law of induction

$$\nabla x\, E = -\frac{\partial B}{\partial t}$$

4. Ampere's law

$$\nabla x\, B = \frac{4\pi k}{c^2} J + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

$$= \frac{J}{\varepsilon_0 c^2} + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

Note:

$$\nabla \cdot E \quad and \quad \nabla x E$$

here represent the vector operations divergence and curl, respectively.

$$k = \frac{1}{4\pi\varepsilon_0} = \text{Coulomb's constant} \qquad c^2 = \frac{1}{\mu_0\varepsilon_0}$$

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Example 1: The magnetic susceptibility of silicon is -0.4×10^{-5}. Calculate the flux density and magnetic moment per unit volume when magnetic field of intensity 5×10^{5} A/m is applied.

Sol.

Example 2: The magnetic field strength in silicon is 1000 A/m. If the magnetic susceptibility is -0.25×10^{-5}, calculate the magnetization and flux density in silicon.

Sol.

Example 3: magnetic field and magnetic intensity are respectively 1.8 T and 1000 A/m. find relative permeability and susceptibility.

Sol.

Example 4: the susceptibility of annealed iron at saturation is 5500. Find the permeability of annealed iron at saturation.

Sol.

Example 5: Derive relation between relative magnetic permeability and magnetic susceptibility.

Sol.

Paramagnetic vs. Diamagnetic

- Paramagnetic materials are attracted by external magnetic fields whereas diamagnetic materials are repelled.
- Paramagnetic materials have at least one unpaired electron in the system, but diamagnetic materials have all their electrons paired.
- The magnetic field created by paramagnetic materials are in the direction of the external magnetic field whereas the magnetic field created by diamagnetic materials are opposing in direction to the external magnetic field.
- Paramagnetism is a stronger magnetic behaviour exhibited only by selective materials, whereas diamagnetism is a weak magnetic behaviour generally shown by all materials and easily suppressed in the presence of stronger magnetic properties.