## Lecture (4)

## Electricity and Magnetism II

Dr. Ahmed Almurshedi Ph.D., M.Sc., B.Sc. Medical Physics

Example: Calculate the magnetic field due to a wire in the shape of an arc that is one sixth of a circle of radius 0.2 m at the center of the circle when it carries a current of 4 A in a clock wise direction. Assume the arc lies on the xy-plane.

Sol.

## Amperes Law for the magnetic field

Let us discuss some of the properties of static or time independent magnetic field.
Consider an infinite rectilinear current (I) the magnetic field $\overrightarrow{\boldsymbol{B}}$ at a point $\mathbf{A}$ is perpendicular to $\mathbf{O A}$ as in the figure below, and given by the equation:

$$
\vec{B}=\frac{\mu_{o} I}{2 \pi r} \overrightarrow{\boldsymbol{u}}_{\theta}
$$

To compute the circulation of $\overrightarrow{\boldsymbol{B}}$ around a circular path of radius $\mathbf{r}$. the magnetic field $\overrightarrow{\boldsymbol{B}}$ is tangent to the path, so that:

$$
\vec{B} \cdot \overrightarrow{d L}=B d L
$$

and is constant in magnitude, therefore the magnetic circulation (designated by $\boldsymbol{\Lambda}_{\boldsymbol{B}}$ and some times called the magnetomotive force $\mathbf{m m f}$ ).

$$
\begin{gathered}
\Lambda_{B}=\oint_{L} \vec{B} \cdot \overrightarrow{d L}=\oint_{L} B d L=B \oint_{L} d L=B L \\
\Lambda_{B}=\frac{\mu_{o} I}{2 \pi r}(2 \pi r)
\end{gathered}
$$



Because $\boldsymbol{L}=\mathbf{2 \pi r}$

$$
\Lambda_{B}=\mu_{o} I
$$

The magnetic circulation $\Lambda_{B}$ is proportional to the electric current ( $\mathbf{I}$ ) and is independent of the path of the radius. Therefore, if around the current (I) we draw several circles $\boldsymbol{L}_{1}, \boldsymbol{L}_{2}, \boldsymbol{L}_{3}, \ldots$ (as in the figure below), the magnetic circulation around all of them is the same, and is equal to ( $\boldsymbol{\mu}_{\boldsymbol{o}} \boldsymbol{I}$ ).


Amperes Law: the circulation of the magnetic field or (mmf) along a closed line which links current $\boldsymbol{I}_{1}, \boldsymbol{I}_{2}, \boldsymbol{I}_{3}, \ldots$ is

$$
\Lambda_{B}=\oint_{L} \vec{B} \cdot \overrightarrow{d L}=\mu_{o} I
$$

Where, $\boldsymbol{I}=\boldsymbol{I}_{\mathbf{1}}+\boldsymbol{I}_{\mathbf{2}}+\boldsymbol{I}_{\mathbf{3}}+\cdots$ stands for the total current linked by the path (L) see figure below.


## Examples on Amperes Law

Example 1: magnetic field produced by a current along a circular cylinder of infinite length. Sol.

Consider a current $\mathbf{I}$ along a cylinder of radius $\mathbf{a}$ (as in the figure).


Figure magnetic field of a cylindrical conductor


Example 2: Magnetic field produced by a toroidal coil.
Sol.
A toroidal coil consists of a wire uniformly wound on a torus, as shown in figure below.


Figure magnetic field of a toroidal coil


Example 3: A toroidal coil with a circumference 0.2 m is wrapped with 1500 turns of wire that carries a current of $\mathbf{1 . 5} \boldsymbol{s}^{\mathbf{- 1}} \boldsymbol{C}$. Find the magnetic field within the toroid.

Example 4: find the magnetic field in air 1 cm from a wire that carries a current of 1A (as in the figure below).
Sol.


