Lecture (3)

Electricity and Magnetism II

First Stage
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Rolling Rod

A rod with a mass **m** and radius **R** is mounted on two parallel rails of length **a** separated by a distance $\boldsymbol{l}$, as in the figure below. The rod carries a current **I** and rolls without slipping along the rails which are placed in a uniform magnetic field $\vec{\boldsymbol{B}}$ directed into the page. If the rod is initially at rest, what is its speed as it leaves the rails?

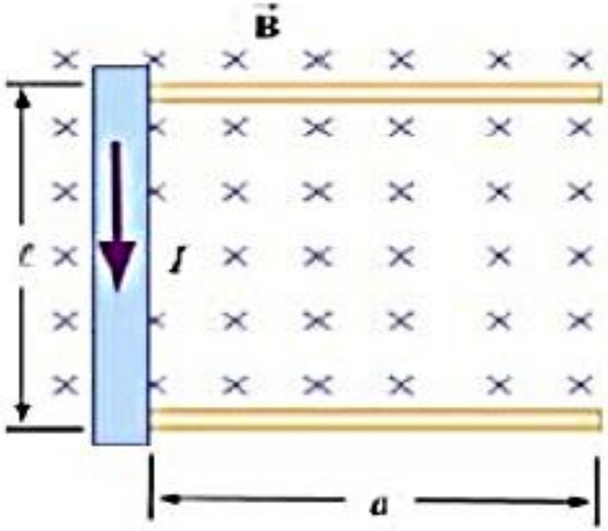

Figure Rolling rod in uniform magnetic field

$$\vec{F}_B = I\,\vec{l} \times \vec{B} = I\,(l\,\acute{i}) \times \left(-B\,\acute{k}\right) = I\,l\,B\,\acute{j}$$

The total work done by the magnetic force on the rod as it moves through the region is

$$W = \int \vec{F}_B . d\vec{s} = \vec{F}_B\,a = I\,l\,B\,a$$

By the work–energy theorem, **W** must be equal to the change in the kinetic energy: $\Delta K = \frac{1}{2}\,m\,v^2 + \frac{1}{2}\,I\,\omega^2$

Where both translation and rolling are involved. Since the moment of inertia of the rod is given by $\boldsymbol{I = \frac{m\,R^2}{2}}$ and the condition of rolling with rolling implies $\boldsymbol{\omega = \frac{v}{R}}$.

$$\boldsymbol{I\,l\,B\,a = \frac{1}{2}\,m\,v^2 + \frac{1}{2}\left(\frac{m\,R^2}{2}\right)\left(\frac{v}{R}\right)^2 = \frac{1}{2}\,m\,v^2 + \frac{1}{4}m\,v^2 = \frac{3}{4}\,m\,v^2}$$

Thus the speed of the rod as it leaves the rails is:

$$\boldsymbol{v = \sqrt{\frac{4\,I\,l\,B\,a}{3\,m}}}$$

Suspended Conducting Rod

A conducting rod having a mass density $\lambda\,kg/m$ is suspended by two flexible wires in a uniform magnetic field $\vec{B}$ which points out of the page as in the figure below.

If the tension of the wire is zero, what are the magnitude and the direction of the current in the rod?

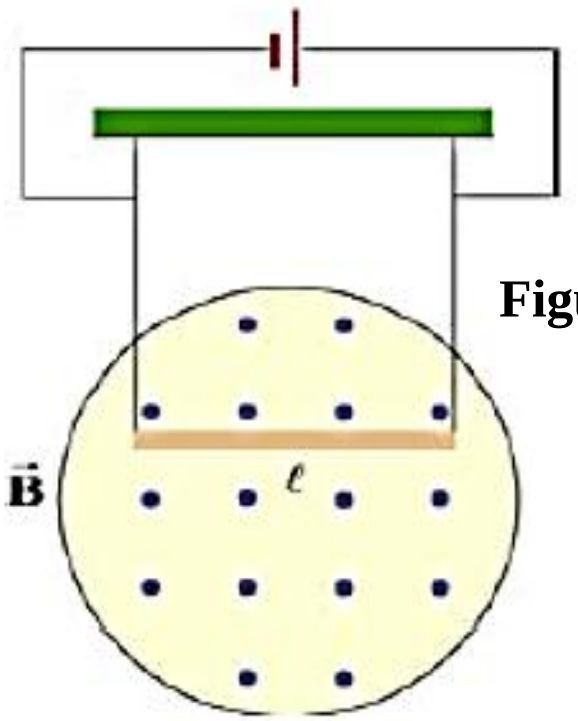

Figure suspended conducting rod in uniform magnetic field

Because of the tension is zero, the magnetic force $\vec{F}_B = I\,\vec{l} \times \vec{B}$ acting on the conductor must exactly cancel the downward gravitational force

$$\vec{F}_s = -mg\,\acute{k}$$

For $\vec{F}_B$ to point in the +z direction we must have $\vec{l} = -l\,\acute{j}$ i.e. the current flows to the left so that:

$$\vec{F}_B = I\,\vec{l} \times \vec{B} = I(-l\,\acute{j}) \times (B\,\acute{i}) = -I\,l\,B\,(\acute{j} \times \acute{i}) = I\,l\,B\,\acute{k}$$

The magnitude of the current can be obtain from

$$I\,l\,B = mg$$

$$I = \frac{m\,g}{l\,B} = \frac{\lambda\,g}{B}$$

Magnetic force on an electric current

- Electric current is a stream of electric charges moving in a vacuum or through matter. The intensity of the electric current has been found as the charge passing per unit time through a section of the conductor.
- Therefore when a conductor carrying an electric current is placed in a magnetic field, it experiences a force which is the resultant of the magnetic forces exerted on each of the moving charges. Then as shown below, the magnetic force on the conductor is giving by:

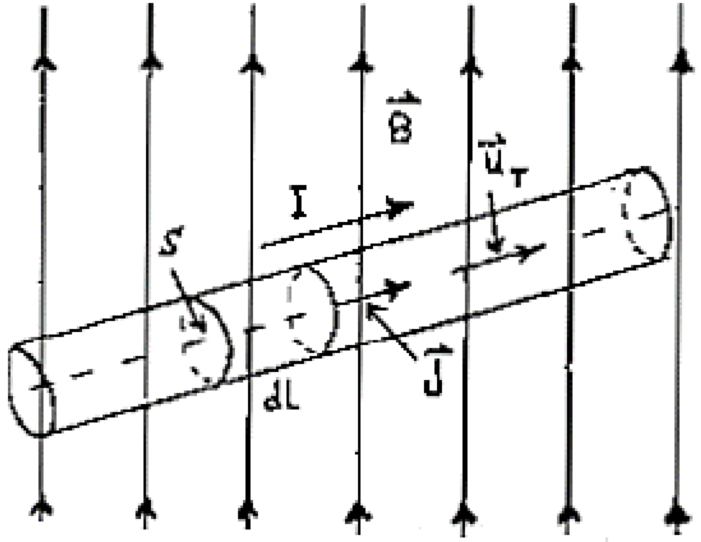

Figure current carrying conductor in a magnetic field

$$\vec{F} = I \int \vec{u}_T \times \vec{B}\, dl$$

In the case of rectilinear conductor placed in a uniform magnetic field $\vec{B}$, both $\vec{u}_T$ *and* $\vec{B}$ are constant we may write:

$$\vec{F} = I\, \vec{u}_T \times \vec{B} \int dl$$

If $L = \int dl$ is the length of the rectilinear conductor, the force is:

$$\vec{F} = I\, L\, \vec{u}_T \times \vec{B}$$

The direction of $\vec{F}$ is perpendicular to $\vec{u}_T$ (is a unit vector along the axis of the filament).

A conductor carrying a current and placed in a magnetic field is subject to a force perpendicular to the current and to the magnetic field.

This is the principle on which electric motors operate. If $\boldsymbol{\theta}$ is the angle between the conductor and the magnetic field, we may write for the magnitude of the force $\vec{\boldsymbol{F}}$.

$$\boldsymbol{F} = \boldsymbol{I}\,\boldsymbol{L}\,\boldsymbol{B}\,\mathbf{sin}\,\boldsymbol{\theta}$$

The force is zero if the conductor is parallel to the field ($\boldsymbol{\theta} = \mathbf{0}$) and maximum if it is perpendicular to it ($\boldsymbol{\theta} = \boldsymbol{\pi}/\mathbf{2}$). The direction of the force is found by applying the right hand rule, as showing in the figure below.

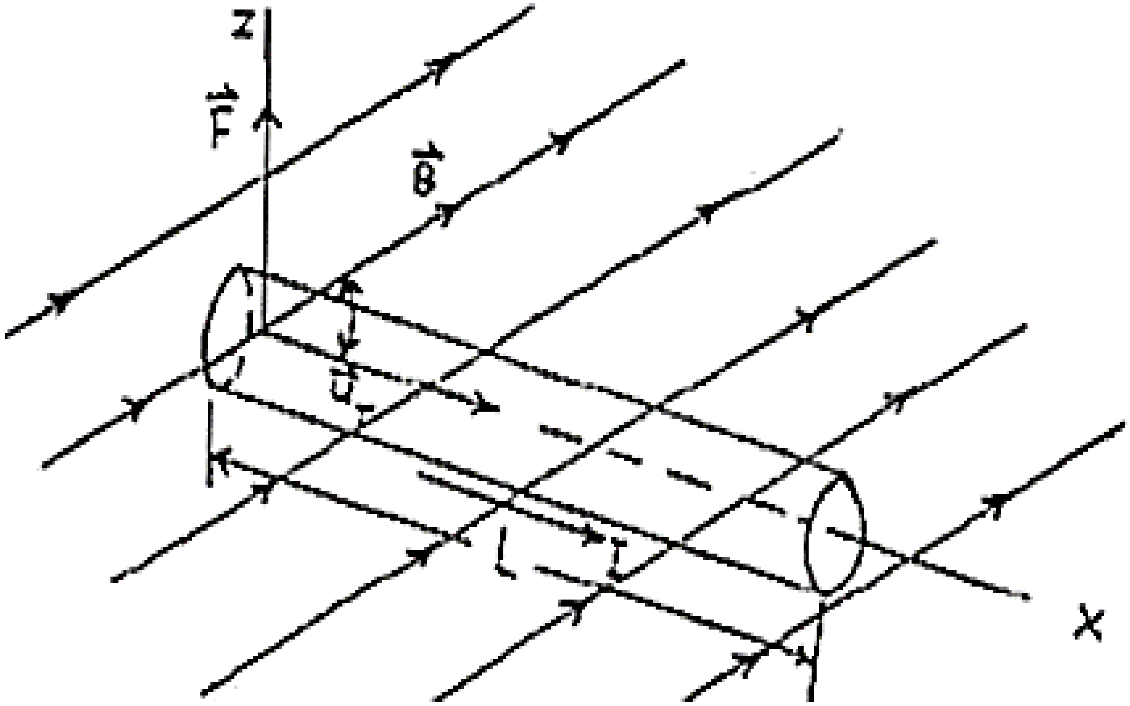

Figure Vector relation between the magnetic force on a current carrying conductor, the magnetic field and the current

Magnetic field produced by a closed current

A general expression has been obtained for calculating the magnetic field produced by a closed current for any shape. This expression called Ampere-Laplace Law: (see figure below).

$$\vec{B} = K_m\, I \oint \frac{\vec{u}_T \times \vec{u}_r}{r^2}\, dL$$

- K_m is a constant depending on the unit chosen
- $K_m = 10^{-7}\, T.m.A^{-1} = 10^{-7} m\, kg\, C^{-2}$

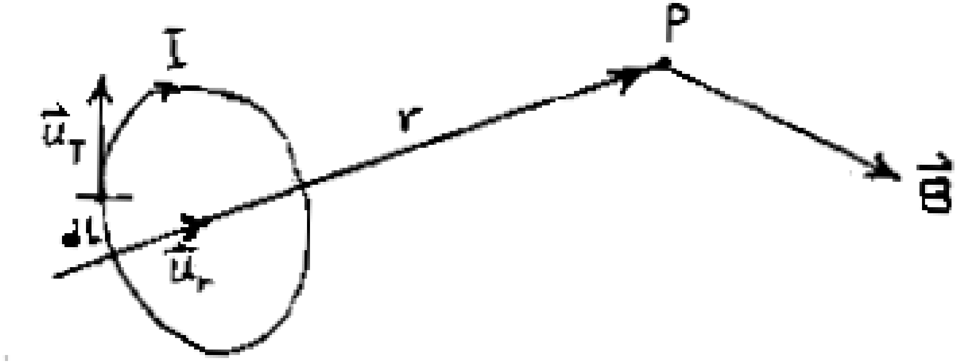

Figure magnetic field produced by an electric current at point P

Also, $K_m = \frac{\mu_o}{4\pi}$

where μ_o is magnetic permeability of vacuum and equals to

$$\mu_o = 4\pi \times 10^{-7}\, m\, kg\, C^{-2}$$

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$\vec{u}_r$ is a unit vector along the radius.

$\vec{u}_T$ is the tangential unit vector.

Above equation for the **Ampere-Laplace law** become:

$$\vec{B} = \frac{\mu_o}{4\pi} I \oint \frac{\vec{u}_T \times \vec{u}_r}{r^2} dL$$

Since an electric current is simply a stream of electric charges moving in the same direction, we come to the important conclusion that “the magnetic field, and accordingly the magnetic interaction, is produced by moving electric charge”

Magnetic field of a rectilinear current (Biot-Savart Law)

Consider a very long and thin rectilinear current as in the figure below.

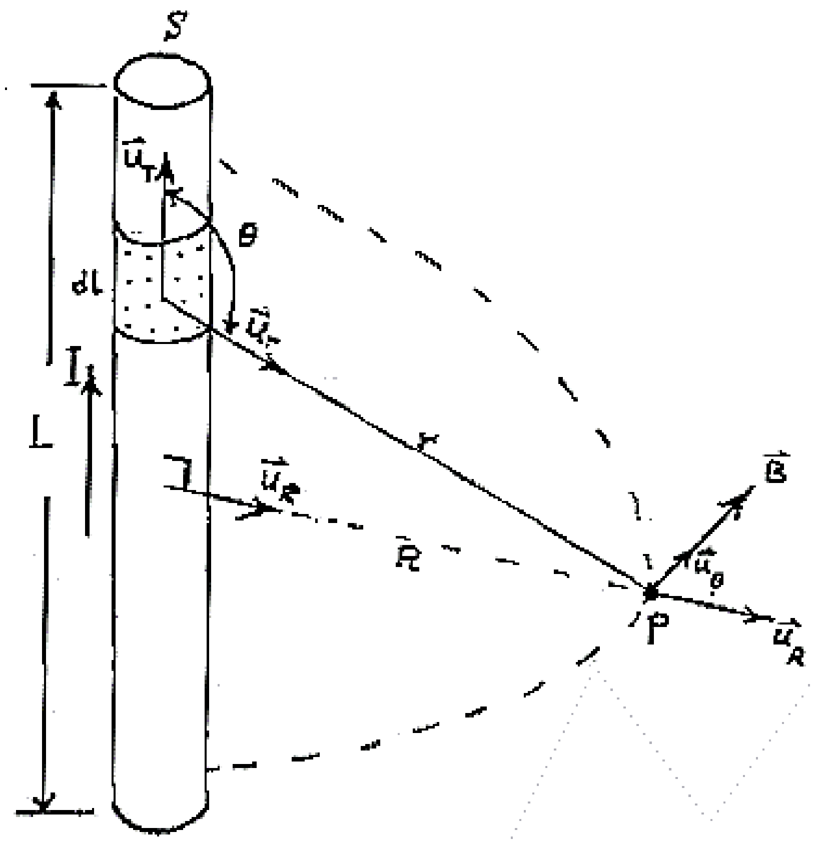

Figure magnetic field produced by a rectilinear current at point P

Application of above equation showing that the magnitude of the magnetic field at a point distance **R** from the current is:

$$\vec{B} = \frac{\mu_o I}{2\pi R}\,\vec{u}_\theta$$

To derive above equation, use the Ampere-Laplace law:

$$\vec{B} = \frac{\mu_o}{4\pi}\,I\oint \frac{\vec{u}_T \times \vec{u}_r}{r^2}\,dL$$

$$\vec{u}_T \times \vec{u}_r = |u_T||u_r|\sin\theta\,\vec{u}_\theta = \sin\theta\,\vec{u}_\theta$$

$$\vec{B} = \frac{\mu_o}{4\pi}\,I\int_{-\infty}^{\infty} \frac{\sin\theta}{r^2}\,dL\,\vec{u}_\theta$$

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In magnitude as

$$B = \frac{\mu_o}{4\pi} I \int_{-\infty}^{\infty} \frac{\sin\theta}{r^2}\, dL$$

From the figure we can see that:

$$r = R\csc\theta \qquad r^2 = R^2\csc^2\theta$$

$$L = -R\cot\theta \qquad dL = R\ \csc^2\theta\ d\theta$$

Subtracting in the above equation and noting that

$$L = -\infty \quad corresponds\ to \quad \theta = 0$$

$$L = +\infty \quad corresponds\ to \quad \theta = \pi$$

We obtain

$$\vec{B} = \frac{\mu_o}{4\pi} I \int_0^{\pi} \frac{\sin\theta}{R^2\csc^2\theta}\left(R\ \csc^2\theta\ d\theta\right) = \frac{\mu_o I}{4\pi R}\int_0^{\pi} \sin\theta\, d\theta$$

$$\vec{B} = \frac{\mu_o I}{4\pi R}\left(-\cos\theta\right)_0^{\pi}$$

Then the magnitude of $\vec{B}$ is:

$$B = \frac{\mu_o I}{2\pi R}$$

and in a vector form as

$$\vec{B} = \frac{\mu_o I}{2\pi R}\vec{u}_\theta$$

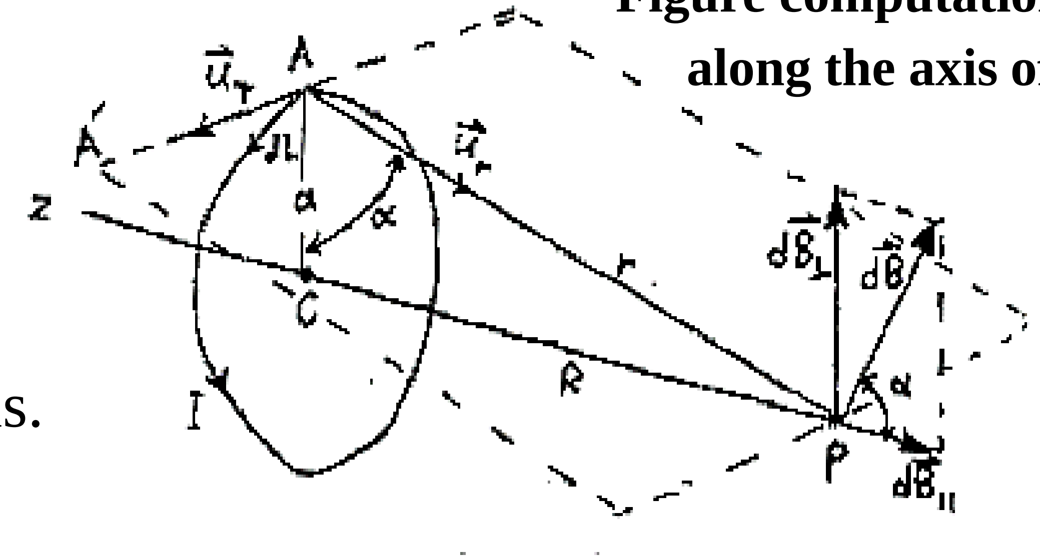
This is called the "**Biot-Savart Law**"

Magnetic field of a circular current

The magnetic field of a circular current is illustrated in figure below, where the vector product $\vec{u}_T \times \vec{u}_r$ is perpendicular to the plane $PA\acute{A}$ and has a magnitude of one because these two unit vectors are perpendicular. Therefore, the field $d\vec{B}$, produced by the length element dL at **P**, has the magnitude:

Figure computation of the magnetic field along the axis of a circular current

$$dB = \frac{\mu_o}{4\pi} I \frac{dL}{r^2}$$



and is perpendicular to the plane $PA\acute{A}$, being thus oblique to the z-axis.

Decomposing $\boldsymbol{d\vec{B}}$ into a component $\boldsymbol{dB_{\parallel}}$ parallel to the axis and a component $\boldsymbol{dB_{\perp}}$ perpendicular to it, we see that, when we integrate along the circle, for each $\boldsymbol{dB_{\perp}}$, there is another in, the opposite direction from the length element directly opposed to $\boldsymbol{dL}$, and therefore all vectors $\boldsymbol{dB_{\perp}}$ add to zero. The resultant **B** will be the sum of all $\boldsymbol{dB_{\parallel}}$, and therefore is parallel to the axis.

Now, since $\boldsymbol{\cos\alpha = a/r}$

$$\boldsymbol{dB_{\parallel} = (\mathrm{dB})\cos\alpha = \frac{a}{r}\, dB = \frac{\mu_o\, I\, a}{4\pi\, r^3}\, dL}$$

The distance **r** remains constant when we integrate around the circle.

$$\boldsymbol{\oint dL = 2\pi\, a}$$

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We may write for the magnitude of the resultant magnetic field

$$\boldsymbol{B} = \oint \boldsymbol{dB_{\parallel}} = \frac{\boldsymbol{\mu_o\, I\, a}}{\boldsymbol{4\pi\, r^3}} \oint \boldsymbol{dL} = \frac{\boldsymbol{\mu_o\, I\, a^2}}{\boldsymbol{2\, r^3}}$$

Noting that: $\boldsymbol{r = (a^2 + R^2)^{1/2}}$

We can write the magnetic field for points on the axis of a circular current as:

$$\boldsymbol{B} = \frac{\boldsymbol{\mu_o\, I\, a^2}}{\boldsymbol{2(a^2 + R^2)^{3/2}}}$$

At **R = 0**, the magnetic field at the center of a circular current is:

$$\boldsymbol{B} = \frac{\boldsymbol{\mu_o\, I}}{\boldsymbol{2\, a}}$$

Magnetic field of moving charge

The fact that an electric current produces a magnetic field suggests that a single moving charge must also produce a magnetic field.

For a charge moving with a velocity $\boldsymbol{v}$, small compared with the velocity of light.

The magnetic field at a point **A**, at a distance **r** from the charge (as in the figure below) is given by:

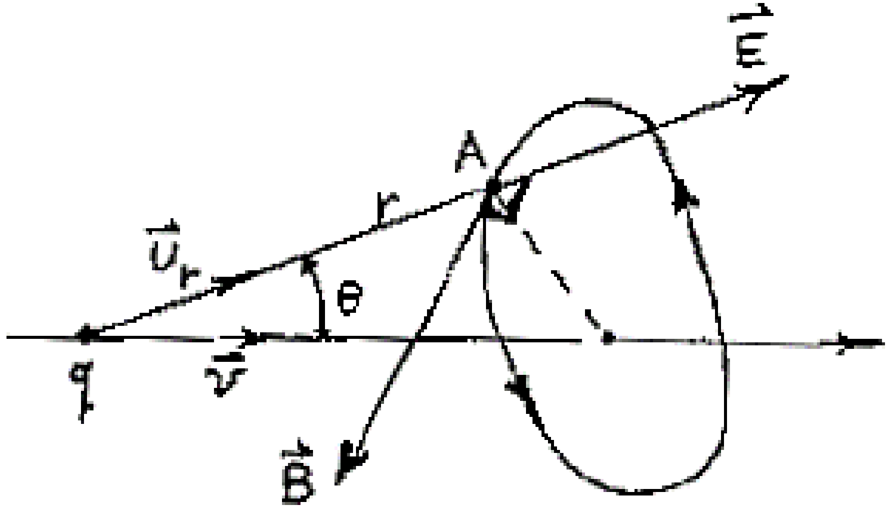

Figure electric and magnetic fields produced by moving charge

$$\vec{B} = \frac{\mu_o}{4\pi} \frac{q\,\vec{v} \times \vec{u}_r}{r^2}$$

and the magnitude of $\vec{B}$ is: $$B = \frac{\mu_o}{4\pi} \frac{q\,v \sin\theta}{r^2}$$

And the direction is perpendicular to $\vec{r}$ and $\vec{v}$. The magnetic lines of force are then circles, θ is the angle between $\vec{v}$ and $\vec{u}_r$.

The magnetic lines of force are then circles, note that the magnitude of the magnetic field is zero along the line of motion, and has its maximum value on the plane perpendicular to the line of motion and passing through the charge. The electric field $\vec{E}$ produced by charge **q** at point **A**,

$$\vec{E} = \frac{q\,\vec{u}_r}{4\,\pi\,\epsilon_o\,r^2}$$

Therefore, we may write: $\vec{B} = \mu_o \epsilon_o\,\vec{v} \times \vec{B} = \frac{1}{c^2}\,\vec{v} \times \vec{E}$

Where, $c = \frac{1}{(\epsilon_o\,\mu_o)^{1/2}} = 3 \times 10^8\,m/s$