one to one (1-1) :- A function $f: D \longrightarrow R$ is one to one iff $a \neq b$ in $D \Longrightarrow f(a) \neq f(b)$

E.X :- $\boxed{f(x) = x^2}$, $x \in D$,

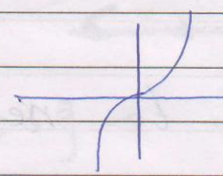
sol;-

$x \in D = (-\infty, \infty)$ is not one to one

$1 \neq -1$ but $f(1) = f(-1) = 1$

هنا اخذنا عددين مختلفين

EX :- $f(x) = x^3$



f is one to one

لان لو اخذنا كل عددين مختلفين مكعبهم تكون مختلفين كذلك -

$1 \neq -1$ but $f(1) \neq f(-1)$

$\therefore f$ is one to one

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Sol ; Domain and Rang

* $y = \sqrt{x^2 - 5x + 6}$

يجب أن يكون ما تحت الجذر ≥ 0

$\sqrt{x^2 - 5x + 6} > 0$

$x^2 - 5x + 6 > 0$

$(x-2)(x-3) > 0$

$\Rightarrow (x-2) > 0 \Rightarrow x > 2$

$(x-3) > 0 \Rightarrow x > 3$

$D = \{x \in \mathbb{R} ; x^2 - 5x + 6 \geq 0\}$

$\therefore D = (-\infty, 2] \cup [3, \infty)$

* $y = \sqrt{x^2 + x + 1}$

$D = \{x \in \mathbb{R} ; x^2 + x + 1 \geq 0\}$

$\therefore D = \mathbb{R}$

* $y = \sqrt{\frac{1+x}{1-x}}$, $D = \{x \in \mathbb{R} ; \frac{1+x}{1-x} \geq 0\}$

$D = [-1, 1)$

لأن المقام لا يساوي صفر

ج Range

$y = \sqrt{-x^2 - 2x + 3}$

نربع الطرفين

$y^2 = -x^2 - 2x + 3$

نصفر المعادلة

$x^2 + 2x - 3 + y^2 = 0$

معامل x^2 $a = 1$

معامل x $b = 2$

الحد المطلق $c = -3 + y^2$

$b^2 - 4ac \geq 0$

$(2)^2 - 4(1)(-3 + y^2) \geq 0$

$4 + 12 - 4y^2 \geq 0$

$16 - 4y^2 \geq 0$

$-4y^2 \geq -16 \quad \div(-4)$

$y^2 \leq 4$ بالجذر

$|y| \leq 2$

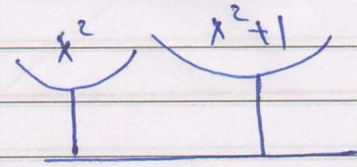
$-2 \leq y \leq 2$

$\therefore$ Rang $y = [-2, 2]$

$$y = \frac{x}{x^2+2}$$

$D = \mathbb{R}$ بالك

$R = ?$



$$0 \leq x^2 < x^2+2$$

$$\frac{0}{x^2+2} \leq \frac{x^2}{x^2+2} < \frac{x^2+2}{x^2+2}$$

نقسم على x^2+2

$$0 \leq \frac{x^2}{x^2+2} < 1$$

$$f(x) = \frac{5}{x-2} \;,$$

Sol :-

$$x - 2 \neq 0 \Rightarrow x \neq 2$$

$$D = (-\infty, 2) \cup (2, \infty)$$

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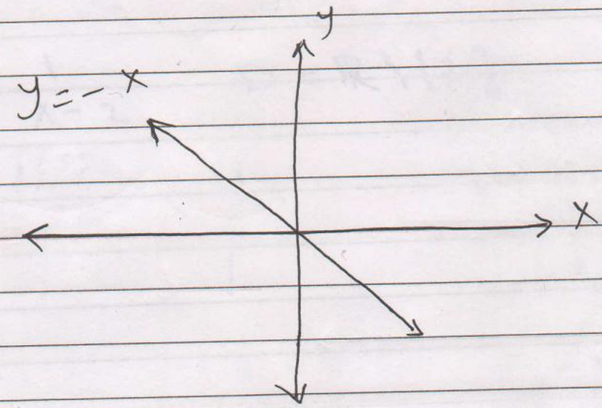
Graphs of function

$f(x) = -x$ $D_f = \mathbb{R} = (-\infty, \infty)$

$R_f = \mathbb{R} = (-\infty, \infty)$



1- $f(x) = x$

Sol :-

x	-2	-1	0	1	2
y	-2	-1	0	1	2

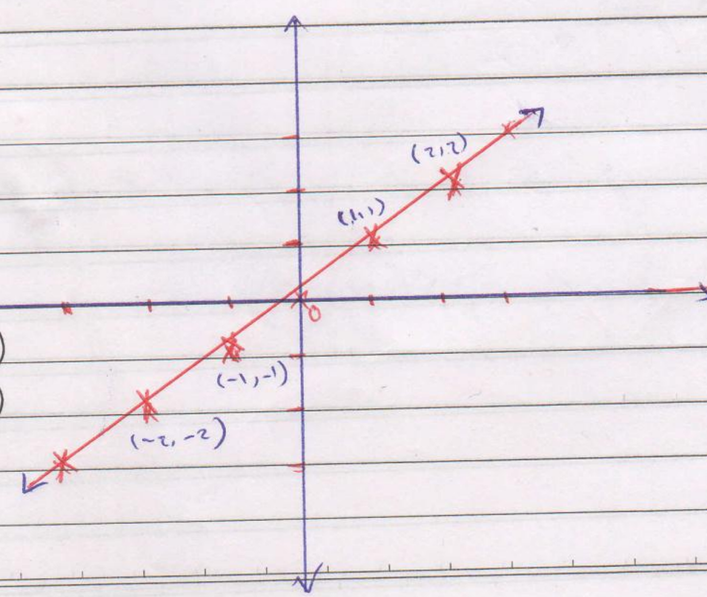
$f(-1) = -1$

$f(-2) = -2$

$f(0) = 0$

$f(1) = 1$

$f(2) = 2$



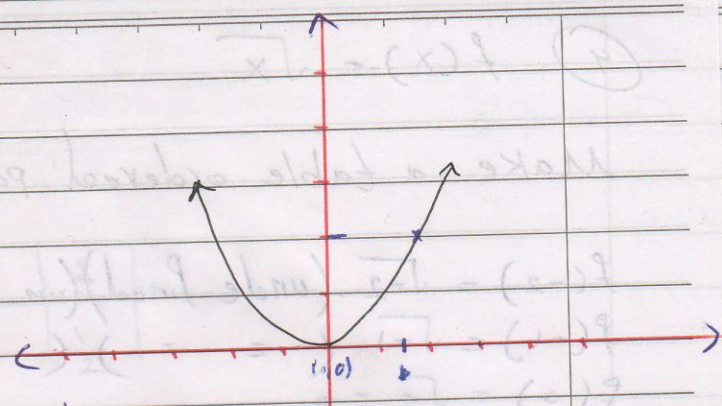
Domain is $(-\infty, \infty)$

Rang is $(-\infty, \infty)$

② $f(x) = x^2$

$f(-2) = (-2)^2 = 4$

$f(-1) = 1$



x	-2	-1	0	1	2
y	4	1	0	1	4

Domain is $(-\infty, \infty)$

Rang is $[0, \infty)$

③ $f(x) = x^3$

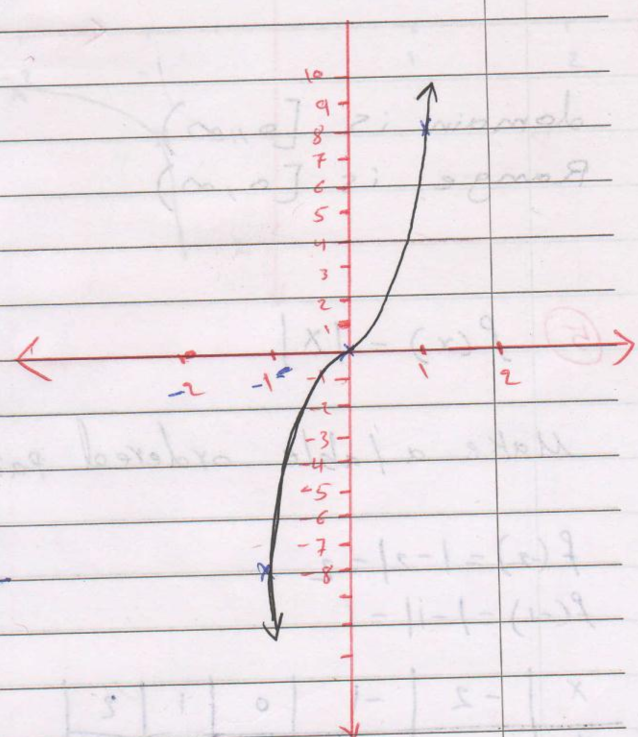
$f(-2) = (-2)^3 = 8$

$f(-1) = -1$

$f(0) = 0$

$f(1) = 1$

$f(2)^3 = 8$



x	-2	-1	0	1	2
y	-8	-1	0	1	8

Domain is $(-\infty, \infty)$

Range is $(-\infty, \infty$

Shifting Graphs

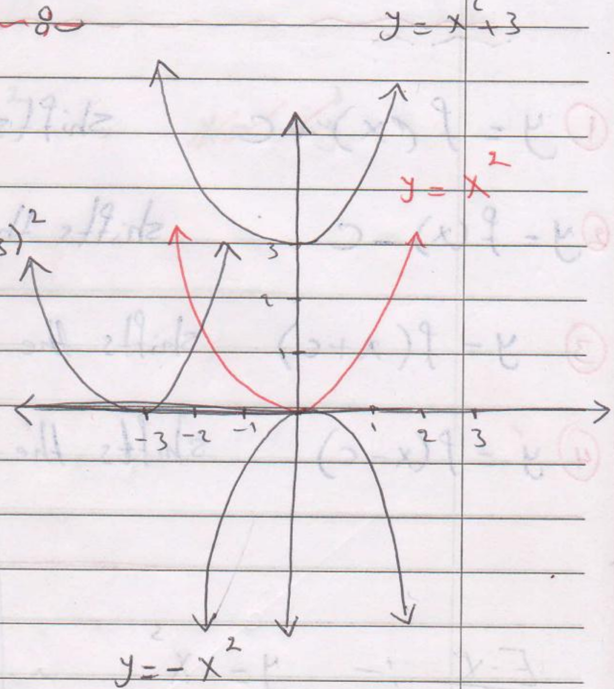
$y = x^2$

$y = -x^2$

$y = x^2 + 3$

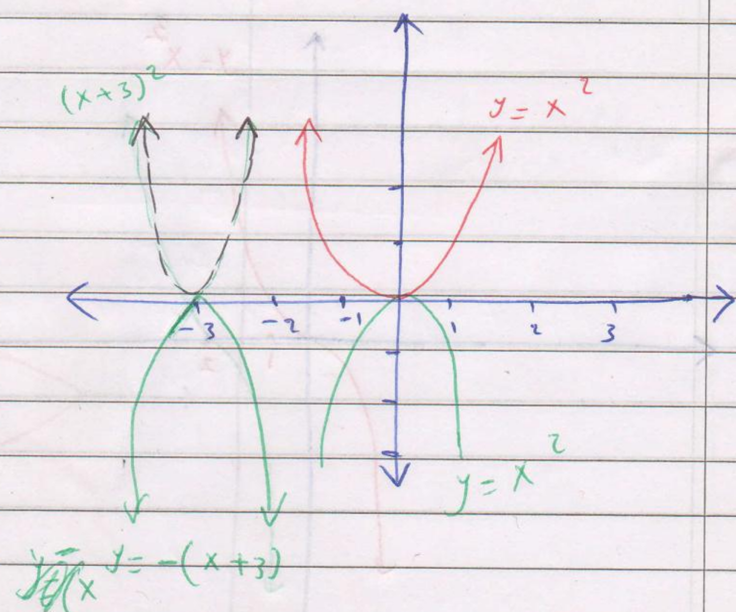
$y = (x+3)^2$

$y = (x-3)^2$



$y = x^2$

$y = -(x+3)^2$



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Shift Formulas $(c > 0)$

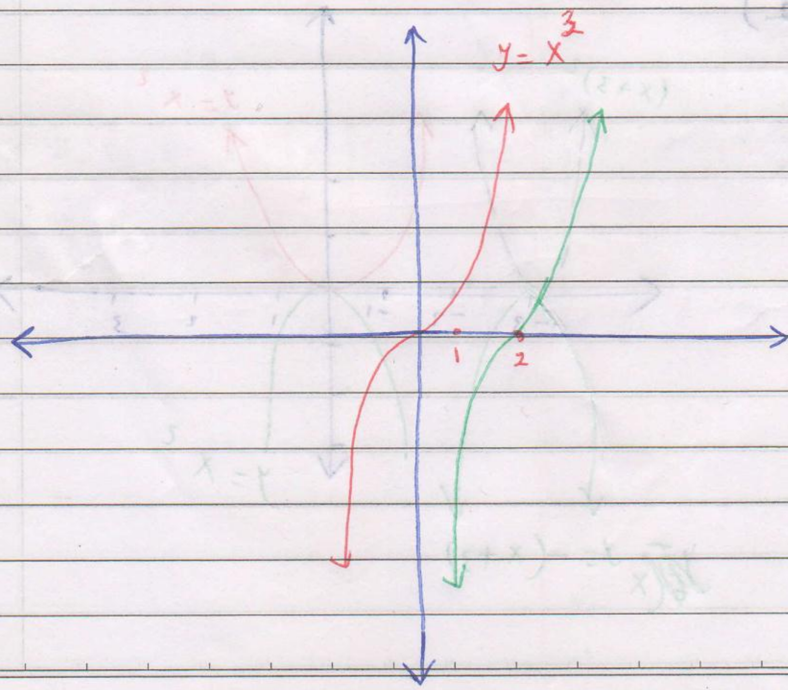
① $y = f(x) + c$ shifts the graph of f up c units

② $y = f(x) - c$ shifts the graph of f down units

③ $y = f(x + c)$ shifts the graph of left c units

④ $y = f(x - c)$ shifts the graph of right c units

E.X :- $y = x^3$, $y = (x-2)^3$



Limits

:- We say that L is a right-hand Limit for $f(x)$ when x approaches c for the right, written $\lim\limits_{x \to c^+} f(x) = L$

Similarly L is the Left-hand limite for $f(x)$ when x apprraches c for the left, written

~~Also~~ $\lim\limits_{x \to c^-} f(x) = L$.

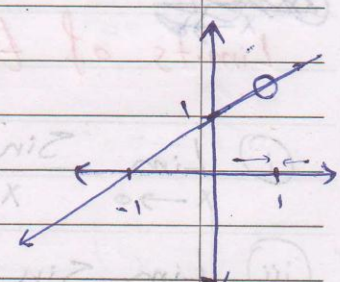
~~then~~.

Then $\lim\limits_{x \to c} f(x) = C$

if and only if $\lim\limits_{x \to c^-} f(x) = \lim\limits_{x \to c^+} f(x)$

Example:

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}$$



$$\lim_{x \to 1} (x+1) = 1 + 1 = \boxed{2}$$

~~Theorem~~ (4) (Some Properties of Limits)

let $f(x)$ and $g(x)$ be two such that both $\lim\limits_{x \to a} f(x)$ and $\lim\limits_{x \to a} g(x)$ exist. Then

① $\lim\limits_{x \to a} [f(x) \mp g(x)] = \lim\limits_{x \to a} f(x) \mp \lim\limits_{x \to a} g(x)$

② For every real number α

$\lim\limits_{x \to a} (\alpha f)(x) = \alpha \lim\limits_{x \to a} f(x)$

③ $\lim\limits_{x \to a} [f(x)\, g(x)] = [\lim\limits_{x \to a} f(x) \cdot \lim\limits_{x \to a} g(x)]$

④ $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x \to a} f(x)}{\lim\limits_{x \to a} g(x)}$, $g(x) \neq 0$

⑤ $\lim\limits_{x \to a} k = k$, k is constant

Limits of trigonometric function :-　　　　مهم

(i) $\lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1$　　(ii) $\lim\limits_{x \to 0} \cos x = 1$

(iii) $\lim\limits_{x \to 0} \sin x = 0$

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~~Example~~

E.x ; (Evaluate)-

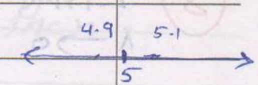
① $\lim_{x\to 2}(4x^2) = 4\lim_{x\to 2} x^2 = 4(2^2) = 4\times 4 = \boxed{16}$

② $\lim_{x\to 0}\frac{x^2+x}{\cancel{x}} = \lim_{x\to 0}\frac{\cancel{x}(x+1)}{\cancel{x}} = \lim_{x\to 0}(x+1) = 1$

③ $\lim_{x\to 2}\left(\frac{x^3-8}{x^2-4}\right) = \lim_{x\to 2}\frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)}$

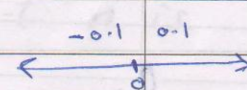
$= \lim_{x\to 2}\frac{x^2+2x+4}{x+2} = \frac{12}{4} = \boxed{3}$

④ $\lim_{x\to 5}\sqrt{2x-6} = \sqrt{10-6} = \sqrt{4} = \boxed{2}$



⑤ $\lim_{x\to 0}\sqrt{x}$ < $\lim_{x\to 0^+}\sqrt{x} = 0$

$\lim_{x\to 0^-}\sqrt{x} =$ غير معرف



∴ $\lim_{x\to 0^+} \neq \lim_{x\to 0^-}$

∴ Lim is not exist

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نضرب ونقسم على 3

(6) $\lim_{x \to 0} \frac{\sin 3x}{x} = 3\lim_{x \to 0} \frac{\sin 3x}{3x} = 3(1) = 3$

(7) $\lim_{x \to 0} \frac{\tan x}{x} = \lim_{x \to 0} \frac{\frac{\sin x}{\cos x}}{x}$

$$= \lim_{x \to 0} \left[\frac{\sin x}{\cos x} \cdot \frac{1}{x} \right]$$

$$= \lim_{x \to 0} \left[\frac{\sin x}{x} \cdot \frac{1}{\cos x} \right]$$

$$= \left[\lim_{x \to 0} \frac{\sin x}{x} \right] \left[\lim_{x \to 0} \frac{1}{\cos x} \right]$$

$$= (1) \cdot (1) = \boxed{1}$$

(8) $\lim_{x \to 0} \frac{|x|}{x}$

$$\frac{x}{x} = 1 \quad x \geq 0$$

$$-\frac{x}{x} = -1 \quad x < 0$$

$$\lim_{x \to 0^+} \frac{x}{x} = 1$$

$$\lim_{x \to 0^-} -\frac{x}{x} = -1$$

$$L^+ \neq L^-$$

السبب انه لو اقتربنا الى الصفر من جهة اليمين فان $f(x)$ هي كمية ثابتة هي 1 اما من ناحية اليسار فهي كمية ثابتة وهي 1-