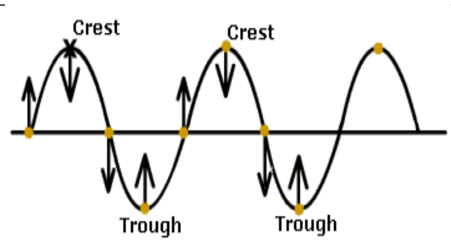
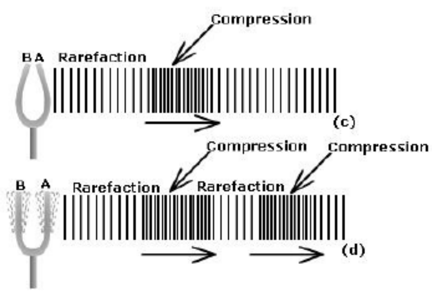
Transverse and Longitudinal Waves

Waves may be transverse, longitudinal, or a combination of the two. Examples of transverse waves are the waves on stringed instruments or surface waves on water. Sound waves in air and water are longitudinal.

Comparison

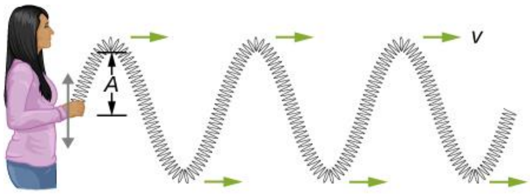
	Transverse waves	Longitudinal waves
Definition	the particles of the medium vibrate **perpendicular** to the direction of the wave propagation	the particles of the medium vibrate **along** to the direction of the wave propagation
Consists of…	The transverse waves are formed of successive ***crests*** and ***troughs***	The longitudinal waves are formed of successive pulses of ***compressions*** and ***rarefactions***
Shape		
Wave length	The distance between two successive crests or troughs	The distance between the centers of two successive compressions or rarefactions
Examples	***Water waves*** And electromagnetic waves radio waves, microwaves, infrared rays, light, ultraviolet rays, X-rays, gamma rays	***Sound waves***

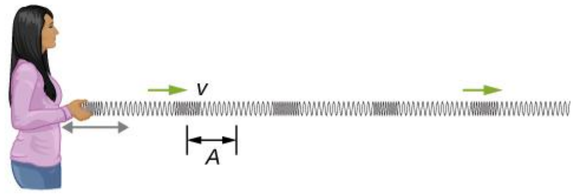
Crests: The maximum displacement of particles of the medium upwards.

Troughs: The maximum displacement of the particles of the medium downwards.

Compressions: Area of high pressure between medium molecules.

Rarefactions: Area of low pressure between medium molecules.





في الشكل (a)، تنتشر الموجة في الاتجاه الأفقي، بينما يكون الوسط مضطربًا في الاتجاه الرأسي. تسمى هذه الموجة بالموجة المستعرضة.

في الموجة المستعرضة، قد تنتشر الموجة في أي اتجاه، ولكن اضطراب الوسط يكون عموديًا على اتجاه الانتشار. وفي المقابل، في الموجة الطولية أو الموجة الانضغاطية، يكون الاضطراب موازيًا لاتجاه الانتشار. يوضح الشكل (b) مثالاً للموجة الطولية.

Example.1// Wave on a String

A student takes a 30 m-long string and attaches one end to the wall in the physics lab. The student then holds the free end of the rope, keeping the tension constant in the rope. The student then begins to send waves down the string by moving the end of the string up and down with a frequency of 2Hz. The maximum displacement of the end of the string is 20 cm. The first wave hits the lab wall 6 s after it was created.

(a) What is the speed of the wave?

(b) What is the period of the wave?

(c) What is the wavelength of the wave?

Solution

a. The first wave traveled 30.00 m in 6.00 s:

$$v = \frac{30.00\ \text{m}}{6.00\ \text{s}} = 5.00\frac{\text{m}}{\text{s}}.$$

b. The period is equal to the inverse of the frequency:

$$T = \frac{1}{f} = \frac{1}{2.00\ \text{s}^{-1}} = 0.50\ \text{s}.$$

c. The wavelength is equal to the velocity times the period:

$$\lambda = vT = 5.00\frac{\text{m}}{\text{s}}(0.50\ \text{s}) = 2.50\ \text{m}.$$

Note/The frequency of the wave produced by an oscillating driving force is equal to the frequency of the driving force.

Example.2// Characteristics of a Wave

A transverse mechanical wave propagates in the positive x-direction through a spring with a constant wave speed, and the medium oscillates between +A and -A around an equilibrium position. The graph in Figure (2) shows the height of the spring (y) versus the position (x), where the x-axis points in the direction of propagation. The figure shows the height of the spring versus the x-position at t=0 s as a dotted line and the wave at t=3s as a solid line. Assume the wave has not traveled more than 1 wavelength in this time.

(a) Determine the wavelength and amplitude of the wave. (b) Find the propagation velocity of the wave. (c) Calculate the period and frequency of the wave.

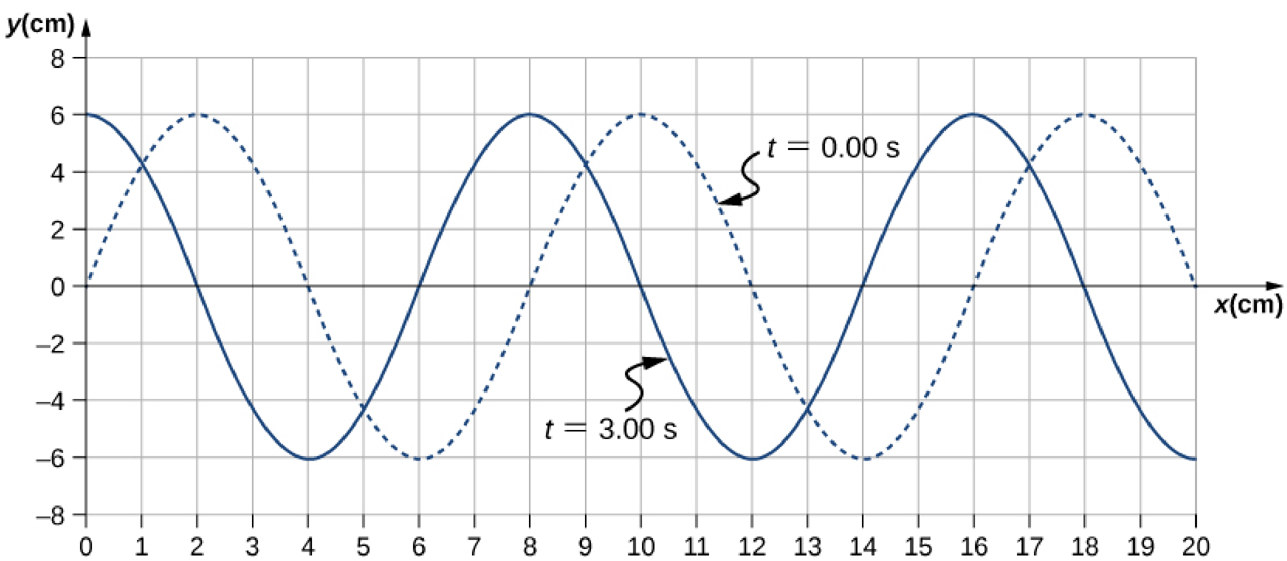


Figure (2)

Solution

a. Read the wavelength from the graph, looking at the purple arrow in Figure. Read the amplitude by looking at the green arrow. The wavelength is λ=3cm and the amplitude is A=6cm .

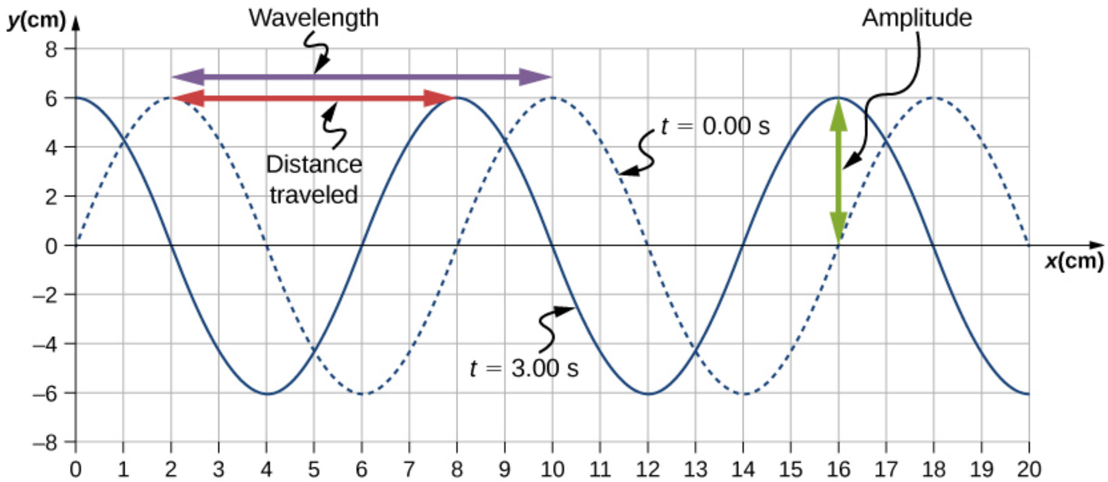


Figure (3) Characteristics of the wave marked on a graph of its displacement.

b. The distance the wave traveled from time t=0 s to time t=3s can be seen in the graph. Consider
the red arrow, which shows the distance the crest has moved in 3 s. The distance is 8 cm-2 cm 6 cm. The velocity is

$$v = \frac{\Delta x}{\Delta t} = \frac{8-2}{3-0} = 2\frac{cm}{s}$$

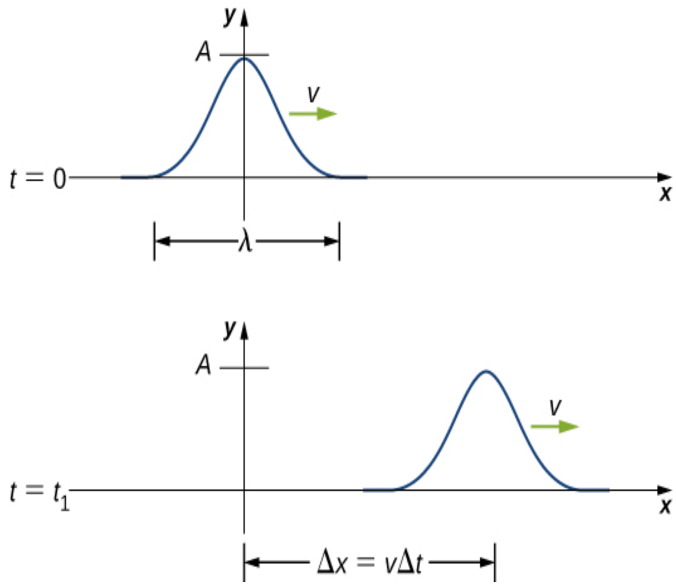
c. The period is $T = \frac{\lambda}{v} = \frac{8}{2}\ 4\ s$ and the frequency is $f = \frac{1}{T} = \frac{1}{4} = 0.25\ Hz$

Mathematics of Waves

The position of particles of the medium can be mathematically modeled as wave functions, which can be used to find the position, velocity, and acceleration of the particles of the medium of the wave at any time.

Pulses

A pulse can be described as wave consisting of a single disturbance that moves through the medium with a constant amplitude. The pulse moves as a pattern that maintains its shape as it propagates with a constant wave speed. Because the wave speed is constant, the distance the pulse moves in a time Δt is equal to Δx=vΔt .



الشكل اعلاه تتمركز النبضة في الوقت t=0 على x=0 بسعة A. وتتحرك النبضة كنمط ذو شكل ثابت، بقيمة قصوى ثابتة A. السرعة ثابتة وتتحرك النبضة مسافة Δx=vΔt في زمن Δt. يتم قياس المسافة المقطوعة بأي نقطة مناسبة على النبض. في هذا الشكل، يتم استخدام القمة.

Modeling a One-Dimensional Sinusoidal Wave using a Wave Function

Consider a string kept at a constant tension FT where one end is fixed and the free end is oscillated between y=+A and y=-A by a mechanical device at a constant frequency.

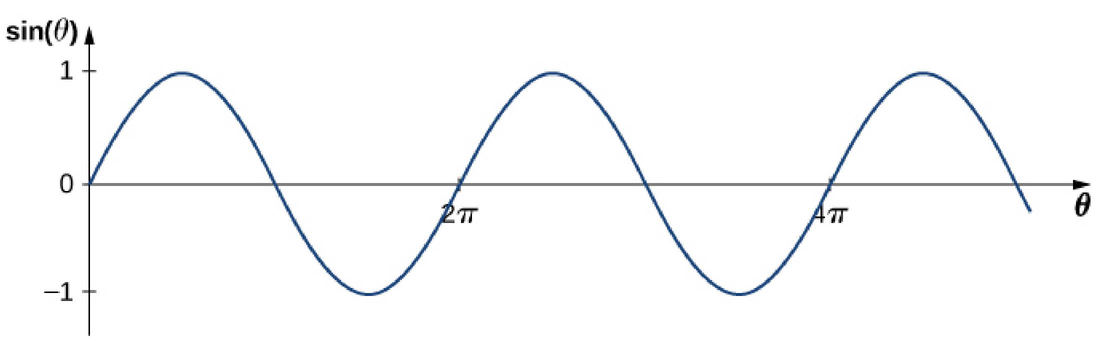


Figure 4 A sine function oscillates between +1 and -1 every 2π radians.

To construct our model of the wave using a periodic function, consider the ratio of the angle and the position,

$$\frac{\theta}{x} = \frac{2\pi}{\lambda},$$
$$\theta = \frac{2\pi}{\lambda}x.$$

Using $\theta = \frac{2\pi}{\lambda}$ and multiplying the sine function by the amplitude A, we can now model the y-position of the string as a function of the position x:

$$y(x) = A\sin\left(\frac{2\pi}{\lambda}x\right)$$

The wave on the string travels in the positive x-direction with a constant velocity v, and moves a distance vt in a time t. The wave function can now be defined by

$$y(x,t) = A\sin\left(\frac{2\pi}{\lambda}(x - vt)\right)$$

It is often convenient to rewrite this wave function in a more compact form. Multiplying through by the ratio leads to the equation

$$y(x,t) = A\sin\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{\lambda}vt\right)$$

The value $\frac{2\pi}{\lambda}$ is defined as the **wave number**. The symbol for the wave number is k and has units of inverse meters,m^{-1}:

$$k \equiv \frac{2\pi}{\lambda} \qquad (2)$$

Recall from Oscillations that the angular frequency is defined as $\omega = \frac{2\pi}{T}$ The second term of the wave function Becomes

$$\frac{2\pi}{\lambda}vt = \frac{2\pi}{\lambda}\left(\frac{\lambda}{T}\right)t = \frac{2\pi}{T}t = \omega t$$

The wave function for a simple harmonic wave on a string reduces to

$$y(x,t) = A\sin(kx \mp \omega t)$$

where A is the amplitude, $k = \frac{2\pi}{\lambda}$ is the wave number, $\omega = \frac{2\pi}{T}$ is the angular frequency, the minus sign is for waves moving in the positive x-direction, and the plus sign is for waves moving in the negative x-direction. The velocity of the wave is equal to

$$v=\frac{\lambda}{T}=\frac{\lambda}{T}\left(\frac{2\pi}{2\pi}\right)=\frac{\omega}{k}$$

The wave function modeling a sinusoidal wave, allowing for an initial phase shift Φ :

$$y(x,t)=A\sin(kx\mp\omega t+\phi)$$

The value $(kx \pm \omega t + \phi)$ is known as the phase of the wave, where ϕ is the initial phase of the wave function. Whether the temporal term ωt is negative or positive depends on the direction of the wave. First consider the minus sign for a wave with an initial phase equal to zero $(\phi = 0)$ The phase of the wave would be Consider following a point on a wave, such as a crest. A crest will occur when $\sin(kx - \omega t) = 1$, that is, when $(kx - \omega t) = n\pi + \frac{\pi}{2}$, for any integral value of n. For instance, one particular crest occurs at $(kx - \omega t) = \frac{\pi}{2}$ .As the wave moves, time increases and x must also increase to keep the phase equal to $\frac{\pi}{2}$. Therefore, the minus sign is for a wave
moving in the positive x-direction. Using the plus sign, $(kx + \omega t) = \frac{\pi}{2}$As time increases, x must decrease to keep the phase equal to $\frac{\pi}{2}$ .The plus sign is used for waves moving in the negative x-direction. In summary,
y(x,t)=A sin $(\boldsymbol{kx - \omega t + \phi})$ models a wave moving in the positive x-direction and
y(x,t)=A sin $(\boldsymbol{kx + \omega t + \phi})$ models a wave moving in the negative x-direction.

Example.3// Characteristics of a Traveling Wave on a String

A transverse wave on a taut string is modeled with the wave function

$$y(x,t)=A\sin(kx-\omega t)=0.2\text{ m}\sin\left(6.28\text{ m}^{-1}x-1.57\text{ s}^{-1}t\right).$$

Find the amplitude, wavelength, period, and speed of the wave.

Solution

1. The amplitude, wave number, and angular frequency can be read directly from the wave equation:

$$y(x,t) = A\sin(kx - wt) = 0.2\text{ m}\sin\left(6.28\text{ m}^{-1}x - 1.57\text{ s}^{-1}t\right).$$

$$(A = 0.2\text{ m}; k = 6.28\text{ m}^{-1}; \omega = 1.57\text{ s}^{-1})$$

2. The wave number can be used to find the wavelength:

$$k = \frac{2\pi}{\lambda}.$$
$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{6.28\text{ m}^{-1}} = 1.0\text{ m}.$$

3. The period of the wave can be found using the angular frequency:

$$\omega = \frac{2\pi}{T}.$$
$$T = \frac{2\pi}{\omega} = \frac{2\pi}{1.57\text{ s}^{-1}} = 4\text{ s}.$$

4. The speed of the wave can be found using the wave number and the angular frequency. The direction of kx ±ωt the wave can be determined by considering the sign of : A negative sign suggests that the wave is moving in the positive x-direction:

$$|v| = \frac{\omega}{k} = \frac{1.57\text{ s}^{-1}}{6.28\text{ m}^{-1}} = 0.25\text{ m/s}.$$

Velocity and Acceleration of the Medium

The velocity of the medium, which is perpendicular to the wave velocity in a transverse wave, can be found by taking the partial derivative of the position equation with respect to time.

$$
\begin{aligned}
y(x,t) &= A\sin(kx-\omega t+\phi) \\
v_y(x,t) &= \frac{\partial y(x,t)}{\partial t} = \frac{\partial}{\partial t}\left(A\sin(kx-\omega t+\phi)\right) \\
&= -A\omega\cos(kx-\omega t+\phi) \\
&= -v_{y\,\max}\cos(kx-\omega t+\phi).
\end{aligned}
$$

The magnitude of the maximum velocity of the medium is $|v_{y_{max}}| = A\omega$. This may look familiar from the Oscillations and a mass on a spring.
We can find the acceleration of the medium by taking the partial derivative of the velocity equation with respect to time,

$$
\begin{aligned}
a_y(x,t) &= \frac{\partial v_y}{\partial t} = \frac{\partial}{\partial t}\left(-A\omega\cos(kx-\omega t+\phi)\right) \\
&= -A\omega^2\sin(kx-\omega t+\phi) \\
&= -a_{y\,\max}\sin(kx-\omega t+\phi).
\end{aligned}
$$

The magnitude of the maximum acceleration is $|a_{y_{max}}| = A\omega^2$.The particles of the medium, or the mass elements, oscillate in simple harmonic motion for a mechanical wave.