Sound Intensity

InWaves, we defined intensity as the power per unit area carried by a wave. Power is the rate at which energy is transferred by the wave. In equation form, intensity I is

$$I = \frac{P}{A} \qquad (8)$$

where P is the power through an area A. The SI unit for I is W/m^2. If we assume that the sound wave is
spherical, and that no energy is lost to thermal processes, the energy of the sound wave is spread over a larger area as distance increases, so the intensity decreases. The area of a sphere is $A = 4\pi r^2$ As the wave spreads out from r_1 to r_2 the energy also spreads out over a larger area:

$$\begin{aligned} P_1 &= P_2 \\ I_1 4\pi r_1^2 &= I_2 4\pi r_2^2; \end{aligned}$$

$$I_2 = I_1 (\frac{r_1}{r_2})^2 \qquad (9)$$

The intensity decreases as the wave moves out from the source. In an inverse square relationship, such as the intensity, when you double the distance, the intensity decreases to one quarter,

$$I_2 = I_1 \left(\frac{r_1}{r_2}\right)^2 = I_1 \left(\frac{r_1}{2r_1}\right)^2 = \frac{1}{4} I_1 .$$

Generally, when considering the intensity of a sound wave, we take the intensity to be the time-averaged value of the power, denoted by $\langle P \rangle$divided by the area,

$$I = \frac{\langle P \rangle}{A} \qquad (10)$$

The intensity of a sound wave is proportional to the change in the pressure squared and inversely proportional to the density and the speed. Consider a parcel of a medium initially undisturbed and then influenced by a sound wave at time t.
The intensity of a sound wave is related to its amplitude squared by:

$$I = \frac{(\Delta p_{max})^2}{2\rho \upsilon} \qquad (11)$$

Here, $\Delta p_{max}$is the pressure variation or pressure amplitude in units of pascals (Pa) or N/m^2. The energy
of an oscillating element of air due to a traveling sound wave is proportional to its amplitude squared. In this equation, ρ is the density of the material in which the sound wave travels, in units of Kg/m^3 and υ is the speed of sound in the medium, in units of m/s.

Human Hearing and Sound Intensity Levels

The ear is a transducer that converts sound waves into electrical nerve impulses in a manner much more sophisticated than, but analogous to, a microphone.
The outer ear, or ear canal, carries sound to the recessed, protected eardrum. The air column in the ear canal resonates and is partially responsible for the sensitivity of the ear to sounds in the 2000–5000-Hz range. The middle ear converts sound into mechanical vibrations and applies these vibrations to the cochlea.
The range of intensities that the human ear can hear depends on the frequency of the sound, but, in general, the range is quite large. The minimum threshold intensity that can be heard is $I_\circ = 10^{-12} W/m^2$ Pain is experienced at intensities of $I_{pain} = 1 W/m^2$Measurements of sound intensity (in units of N/m^2) are very cumbersome due to this large range in values. For this reason, as well as for other reasons, the concept of sound intensity level was proposed.
The **sound intensity level** of a sound, measured in decibels, having an intensity I in watts per meter squared, is defined as

$$\beta(dB) = 10\log_{10}\left(\frac{1}{I_\circ}\right) \qquad (12)$$

Example//
Calculate the sound intensity level in decibels for a sound wave traveling in air at 0C and having a pressure
amplitude of 0.656 Pa.
solution//

1. Identify knowns:
   Sound travels at 331 m/s in air at 0°C.
   Air has a density of 1.29 kg/m^3 at atmospheric pressure and 0°C.
2. Enter these values and the pressure amplitude into $I = \frac{(\Delta p)^2}{2\rho v}$.

$$I = \frac{(\Delta p)^2}{2\rho v} = \frac{(0.656\ \text{Pa})^2}{2(1.29\ \text{kg/m}^3)(331\ \text{m/s})} = 5.04 \times 10^{-4}\ \text{W/m}^2.$$

3. Enter the value for I and the known value for I_0 into $\beta(\text{dB}) = 10\log_{10}(I/I_0)$. Calculate to find the sound intensity level in decibels:

$$10\log_{10}(5.04 \times 10^{8}) = 10(8.70)\text{dB} = 87\ \text{dB}.$$

Example//
Show that if one sound is twice as intense as another, it has a sound level about 3 dB higher
Solution//

1. Identify knowns:
   The ratio of the two intensities is 2 to 1, or
   $$\frac{I_2}{I_1} = 2.00.$$
   We wish to show that the difference in sound levels is about 3 dB. That is, we want to show:
   $$\beta_2 - \beta_1 = 3 \text{ dB}.$$
   Note that
   $$\log_{10} b - \log_{10} a = \log_{10}\left(\frac{b}{a}\right).$$
2. Use the definition of β to obtain
   $$\beta_2 - \beta_1 = 10 \log_{10}\left(\frac{I_2}{I_1}\right) = 10 \log_{10} 2.00 = 10(0.301) \text{ dB}.$$
   Thus,
   $$\beta_2 - \beta_1 = 3.01 \text{ dB}.$$

Echoes

Echoes arc produced by the reflection of sound from a hard surface such as a wall or a mountain cliff. Let us suppose that a person claps his hands when standing at some distance away from a high wall and then listens to the echo. The time that elapses before the echo arrives, will depend on the distance of the wall from the observer. An echo occurs when the reflected sound wave returns to the observer 0.1 second or more after the original wave had reached him so that a distinct repetition of the original sound perceived**.**

Normal Modes of a Standing Sound Wave

Interference of Sound Waves

In Waves, We found that the wave function resulting from the superposition of $y_1(x,t) = A\sin(kx - \omega t + \phi)$ and $y_2(x,t) = A\sin(kx - \omega t)$ is :

$$y(x,t) = \left[2A\cos\left(\frac{\phi}{2}\right)\right]\sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

The phase difference at each point is due to the different path lengths traveled by each wave. When the
difference in the path lengths is an integer multiple of a wavelength,

$$\Delta r = |r_2 - r_1| = n\lambda, \text{ where } n = 0, 1, 2, 3, \ldots,$$

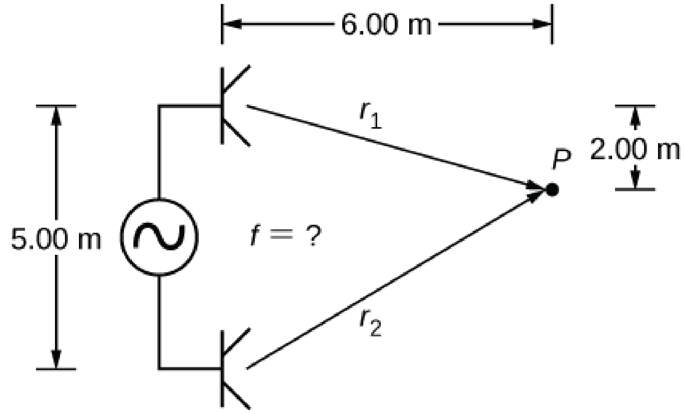
the waves are in phase and there is constructive interference. When the difference in path lengths is an odd multiple of a half wavelength,

$$\Delta r = |r_2 - r_1| = n\frac{\lambda}{2}, \text{ where } n = 1, 3, 5, \ldots,$$

the waves are $180(\pi rad)$ out of phase and the result is destructive interference. These points can be located with a sound-level intensity meter.

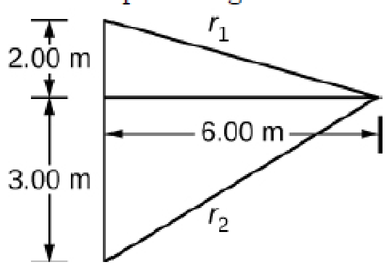
Example// Interference of Sound Waves

Two speakers are separated by 5.00 m and are being driven by a signal generator at an unknown frequency. A student with a sound-level meter walks out 6.00 m and down 2.00 m, and finds the first minimum intensity, as shown below. What is the frequency supplied by the signal generator? Assume the wave speed of sound is v=343 m/s.



Solution//

1. Find the path length to the minimum point from each speaker.



$$r_1 = \sqrt{(6.00\text{ m})^2 + (2.00\text{ m})^2} = 6.32\text{ m},\ \ r_2 = \sqrt{(6.00\text{ m})^2 + (3.00\text{ m})^2} = 6.71\text{ m}$$

2. Use the difference in the path length to find the wavelength.

$$\Delta r = |r_2 - r_1| = |6.71\text{ m} - 6.32\text{ m}| = 0.39\text{ m}$$

$$\lambda = 2\Delta r = 2\,(0.39\text{ m}) = 0.78\text{ m}$$

3. Find the frequency.

$$f = \frac{v}{\lambda} = \frac{343.00\text{ m/s}}{0.78\text{ m}} = 439.74\text{ Hz}$$

Beats

An interesting phenomenon that occurs due to the constructive and destructive interference of two or more frequencies of sound is the phenomenon of beats. If two sounds differ in frequencies, the sound waves can be modeled as

$$y_1 = A\cos(k_1 x - 2\pi f_1 t) \text{ and } y_2 = A\cos(k_2 x - 2\pi f_2 t).$$

Using the trigonometric identity $\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$ and considering the point in space as $x = 0.0$ m, we find the resulting sound at a point in space, from the superposition of the two sound waves, is equal:

$$y(t) = 2A\cos(2\pi f_{avg} t)\cos\left(2\pi\left(\frac{|f_2 - f_1|}{2}\right)t\right),$$

where the **beat frequency** is

$$f_{beat} = |f_2 - f_1| \qquad (17)$$

These beats can be used by piano tuners to tune a piano. A tuning fork is struck and a note is played on the piano. As the piano tuner tunes the string, the beats have a lower frequency as the frequency of the note played approaches the frequency of the tuning fork.

The Doppler Effect

The Doppler effect is an alteration in the observed frequency of a sound due to motion of either the source or the observer. Although less familiar, this effect is easily noticed for a stationary source and moving observer.

For example, if you ride a train past a stationary warning horn, you will hear the horn's frequency shift from high to low as you pass by. The actual change in frequency due to relative motion of source and observer is called a Doppler shift. The Doppler effect and Doppler shift are named for the Austrian physicist and mathematician Christian Johann Doppler (1803–1853), who did experiments with both moving sources and moving observers.

Derivation of the Observed Frequency due to the Doppler Shift.

Consider two stationary observers X and Y in Figure.30 , located on either side of a stationary source. Each observer hears the same frequency, and that frequency is the frequency produced by the stationary source.

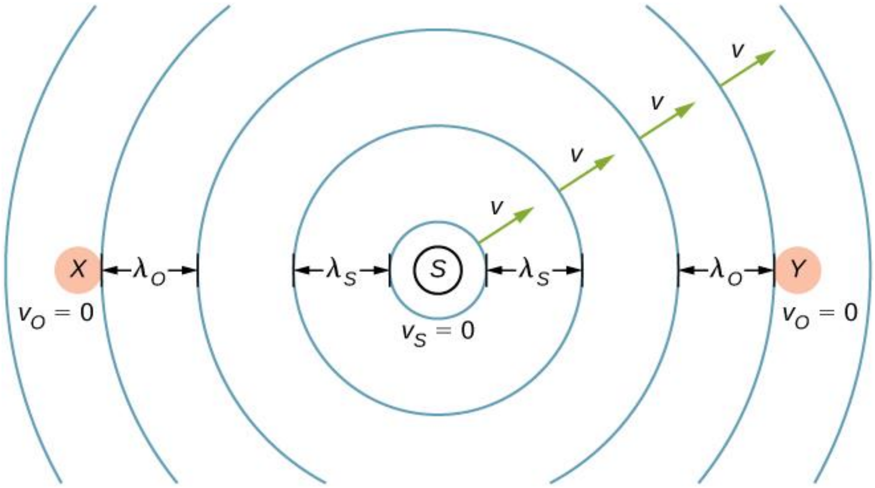


Figure.30

Using the fact that the wavelength is equal to the speed times the period, and the period is the inverse of the frequency, we can derive the observed frequency:

$$
\begin{aligned}
\lambda_o &= \lambda_s + \Delta x \\
vT_o &= vT_s + v_s T_s \\
\frac{v}{f_o} &= \frac{v}{f_s} = \frac{v_s}{f_s} = \frac{v+v_s}{f_s} \\
f_o &= f_s\left(\frac{v}{v+v_s}\right).
\end{aligned}
$$

As the source moves away from the observer, the observed frequency is lower than the source frequency. Now consider a source moving at a constant velocity v_s moving toward a stationary observer Y, The wavelength is observed by Y as $\lambda_\circ = \lambda_s - \Delta x = \lambda_s - v_s T_s$ Once again, using the fact that the wavelength is equal to the speed times the period, and the period is the inverse of the frequency, we can derive the observed frequency:

$$
\begin{aligned}
\lambda_o &= \lambda_s - \Delta x \\
vT_o &= vT_s - v_s T_s \\
\frac{v}{f_o} &= \frac{v}{f_s} - \frac{v_s}{f_s} = \frac{v-v_s}{f_s} \\
f_o &= f_s\left(\frac{v}{v-v_s}\right).
\end{aligned}
$$

When a source is moving and the observer is stationary, the observed frequency is

$$f_\circ = f_s\left(\frac{v}{v \mp v_s}\right) \qquad (18)$$

where f is the frequency observed by the stationary observer, f is the frequency produced by the moving source, v is the speed of sound, $v_s$is the constant speed of the source, and the top sign is for the source approaching the observer and the bottom sign is for the source departing from the observer.

If the observer is moving away from the source (Figure 31), the observed frequency can be found:

$$\begin{aligned}
\lambda_s &= vT_o - v_oT_o \\
vT_s &= (v - v_o)T_o \\
v\left(\frac{1}{f_s}\right) &= (v - v_o)\left(\frac{1}{f_o}\right) \\
f_o &= f_s\left(\frac{v-v_o}{v}\right).
\end{aligned}$$

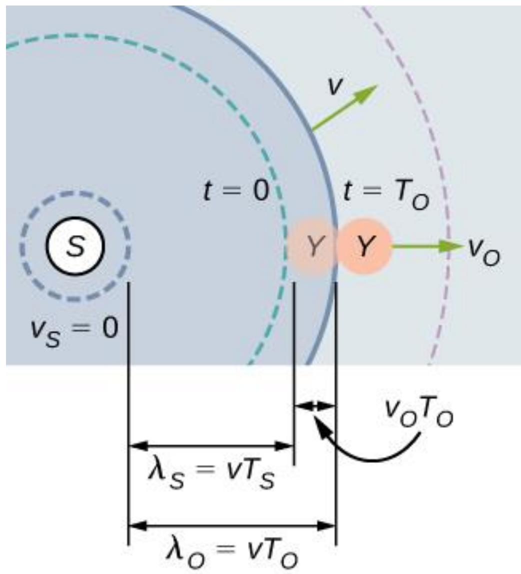


Figure.31: A stationary source emits a sound wave with a constant frequency .

The equations for an observer moving toward or away from a stationary source can be combined into one equation:

$$f_\circ = f_s\left(\frac{\upsilon \pm \upsilon_\circ}{\upsilon}\right) \qquad (19)$$

Where $f_\circ$ is the observed frequency, f_s is the source frequency, υ is the speed of sound, $\upsilon_\circ$is the speed of the observer, the top sign is for the observer approaching the source and the bottom sign is for the observer departing from the source.

Equation .18 and Equation.19 can be summarized in one equation (the top sign is for approaching) and is further illustrated in Table:

$$f_\circ = f_s\left(\frac{\upsilon \pm \upsilon_\circ}{\upsilon \pm \upsilon_s}\right) \qquad (20)$$

Doppler shift $f_o = f_s\left(\frac{v \pm v_o}{v \mp v_s}\right)$	Stationary observer	Observer moving towards source	Observer moving away from source
Stationary source	$f_o = f_s$	$f_o = f_s\left(\frac{v+v_o}{v}\right)$	$f_o = f_s\left(\frac{v-v_o}{v}\right)$
Source moving towards observer	$f_o = f_s\left(\frac{v}{v-v_s}\right)$	$f_o = f_s\left(\frac{v+v_o}{v-v_s}\right)$	$f_o = f_s\left(\frac{v-v_o}{v-v_s}\right)$
Source moving away from observer	$f_o = f_s\left(\frac{v}{v+v_s}\right)$	$f_o = f_s\left(\frac{v+v_o}{v+v_s}\right)$	$f_o = f_s\left(\frac{v-v_o}{v+v_s}\right)$

Example// Calculating a Doppler Shift

Suppose a train that has a 150-Hz horn is moving at 35.0 m/s in still air on a day when the speed of sound is 340 m/s.

(a) What frequencies are observed by a stationary person at the side of the tracks as the train approaches and after it passes?

(b) What frequency is observed by the train's engineer traveling on the train?

Solution//

a. Enter known values into $f_o = f_s\left(\frac{v}{v-v_s}\right)$:

$$f_o = f_s\left(\frac{v}{v - v_s}\right) = (150\text{ Hz})\left(\frac{340\text{ m/s}}{340\text{ m/s} - 35.0\text{ m/s}}\right).$$

Calculate the frequency observed by a stationary person as the train approaches:

$$f_o = (150\text{ Hz})(1.11) = 167\text{ Hz}.$$

Use the same equation with the plus sign to find the frequency heard by a stationary person as the train recedes:

$$f_o = f_s\left(\frac{v}{v + v_s}\right) = (150\text{ Hz})\left(\frac{340\text{ m/s}}{340\text{ m/s} + 35.0\text{ m/s}}\right).$$

Calculate the second frequency:

$$f_o = (150\text{ Hz})(0.907) = 136\text{ Hz}.$$

b. Identify knowns:
   - It seems reasonable that the engineer would receive the same frequency as emitted by the horn, because the relative velocity between them is zero.
   - Relative to the medium (air), the speeds are $v_s = v_o = 35.0$ m/s.
   - The first Doppler shift is for the moving observer; the second is for the moving source.

Use the following equation:

$$f_o = \left[f_s\left(\frac{v \pm v_o}{v}\right)\right]\left(\frac{v}{v \mp v_s}\right).$$

The quantity in the square brackets is the Doppler-shifted frequency due to a moving observer. The factor
on the right is the effect of the moving source.
Because the train engineer is moving in the direction toward the horn, we must use the plus sign for v_{obs};
however, because the horn is also moving in the direction away from the engineer, we also use the plus
sign for v_s But the train is carrying both the engineer and the horn at the same velocity, so $v_s = v_\circ$ As a
result, everything but f_s cancels, yielding $f_s = f_\circ$.

shock wave

When discussing the Doppler effect of a moving source and a stationary observer, the only cases we considered were cases where the source was moving at speeds that were less than the speed of sound. Recall that the observed frequency for a moving source approaching a stationary observer is $f_\circ = f_s\left(\frac{v}{v-v_s}\right)$ As the source approaches the speed of sound, the observed frequency increases. According to the equation, if the source moves at the speed of sound, the denominator is equal to zero, implying the observed frequency is infinite. If the source moves at speeds greater than the speed of sound, the observed frequency is negative.

What happens when a source approaches the speed of sound? It was once argued by some scientists that such a large pressure wave would result from the constructive interference of the sound waves, that it would be impossible for a plane to exceed the speed of sound because the pressures would be great enough to destroy the airplane. But now planes routinely fly faster than the speed of sound.

If the source exceeds the speed of sound, no sound is received by the observer until the source has passed, so that the sounds from the approaching source are mixed with those from it when receding. This mixing appears messy, but something interesting happens—a **shock wave** is created.