

Sound Waves

The physical phenomenon of sound is a disturbance of matter that is transmitted from its source outward.

Hearing is the perception of sound, just as seeing is the perception of visible light. On the atomic scale, sound is a disturbance of atoms that is far more ordered than their thermal motions. In many instances, sound is a periodic wave, and the atoms undergo simple harmonic motion. Thus, sound waves can induce oscillations and resonance effects.

The speaker vibrates at a constant frequency and amplitude, producing vibrations in the surrounding air molecules. As the speaker oscillates back and forth, it transfers energy to the air, mostly as thermal energy. But a small part of the speaker's energy goes into compressing and expanding the surrounding air, creating slightly higher and lower local pressures. These compressions (high-pressure regions) and rarefactions (low-pressure regions) move out as longitudinal pressure waves having the same frequency as the speaker—they are the disturbance that is a sound wave. (Sound waves in air and most fluids are longitudinal, because fluids have almost no shear strength. In solids, sound waves can be both transverse and longitudinal.)

HP = Compression LP = Rarefaction

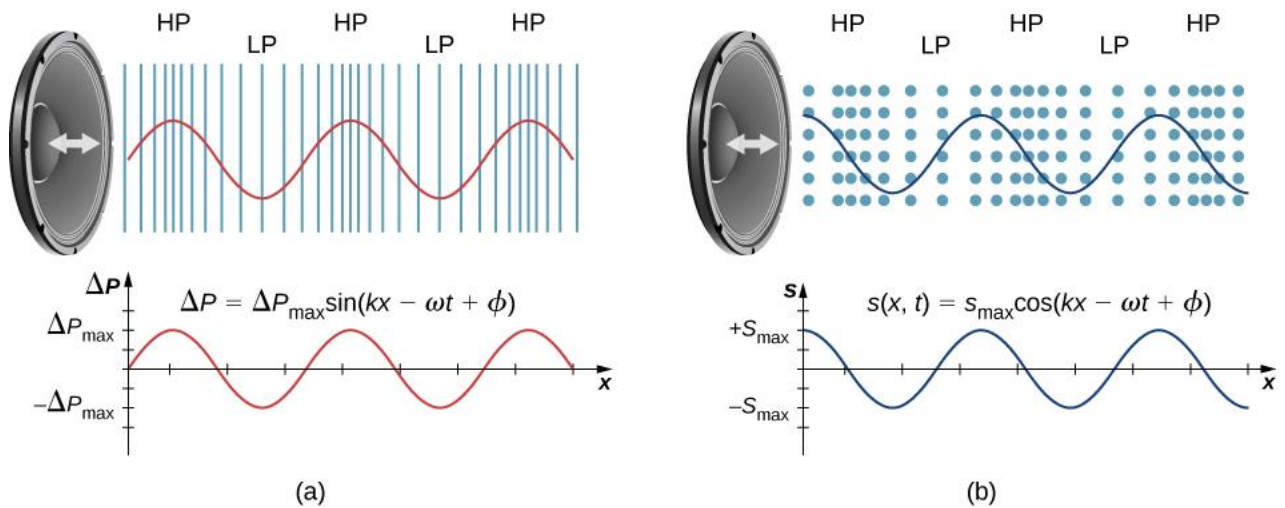


Figure.25: (a) A vibrating cone of a speaker, moving in the positive x-direction, compresses the air in front of it and expands the air behind it. (b) Sound waves can also be modeled using the displacement of the air molecules.

Types of Sound Waves

The three basic types of sound waves are classified based on whether they are audible to humans. Audible sound waves are those sound waves that humans can hear. Infrasonic waves are sound waves that are too low-frequency for humans to hear. And ultrasonic sounds waves are sound waves that are too high-frequency for humans to hear.

In general, the human eardrums can detect sound waves with a frequency between 20 Hertz and 20,000 Hertz. Thus, any sound wave below 20 Hertz (i.e. a wave generated by an

earthquake) is classified as infrasonic. Any sound above 20,000 Hertz (i.e. a wave generated by a dog whistle) is classified as ultrasonic. However, the vast majority of sound waves, from those generated by our voices to those made by musical instruments, fall between 20 and 20,000 Hertz and are therefore audible.

In general, infrasonic waves are not used by humans. Audible sound waves are used all the time, in everything from our speech, to our TV shows, to alarm systems, to music

Models Describing Sound

Sound can be modeled as a pressure wave by considering the change in pressure from average pressure,

$$\Delta P = \Delta P_{max} \sin(kx \mp \omega t + \phi) \quad (1)$$

This equation is similar to the periodic wave equations seen in Waves, where ΔP is the change in pressure,

ΔP_{max} is the maximum change in pressure, $k = \frac{2\pi}{\lambda}$ is the wave number, $\omega = \frac{2\pi}{T} = 2\pi f$ is the angular

frequency, and ϕ is the initial phase. The wave speed can be determined from $v = \frac{\omega}{k} = \frac{\lambda}{T}$. Sound waves

can also be modeled in terms of the displacement of the air molecules. The displacement of the air

molecules can be modeled using a cosine function:

$$s(x, t) = s_{max} \cos(kx \mp \omega t + \phi) \quad (2)$$

In this equation, s is the displacement and s_{max} is the maximum displacement.

لا يظهر في الشكل سعة الموجة الصوتية لأنها تتناقص مع المسافة من مصدرها، لأن طاقة الموجة تنتشر على مساحة أكبر فأكبر. تنخفض الشدة كلما ابتعدت عن مكبر الصوت. تمتص الأجسام الطاقة أيضًا وتحويلها إلى طاقة حرارية عن طريق لزوجة الهواء. بالإضافة إلى ذلك، خلال كل ضغط، يتم نقل القليل من الحرارة إلى الهواء؛ خلال كل تخلخل، يتم نقل حرارة أقل من الهواء، وتؤدي عمليات نقل الحرارة هذه إلى تقليل الاضطراب المنظم إلى حركات حرارية عشوائية. ما إذا كان انتقال الحرارة من الانضغاط إلى التخلخل مهمًا يعتمد على مدى تباعدهما، أي أنه يعتمد على الطول الموجي. يعد الطول الموجي والتردد والسعة وسرعة الانتشار من الخصائص المهمة للصوت، كما هو الحال بالنسبة لجميع الموجات.

Speed of Sound

Sound, like all waves, travels at a certain speed and has the properties of frequency and wavelength. You see the flash of an explosion well before you hear its sound and possibly feel the pressure wave, implying both that sound travels at a finite speed and that it is much slower than light.

The difference between the speed of light and the speed of sound can also be experienced during an electrical storm. The flash of lighting is often seen before the clap of thunder.

The velocity of any wave is related to its frequency and wavelength by

$$v = \lambda f \quad (3)$$

where v is the speed of the wave, f is its frequency, and λ is its wavelength.

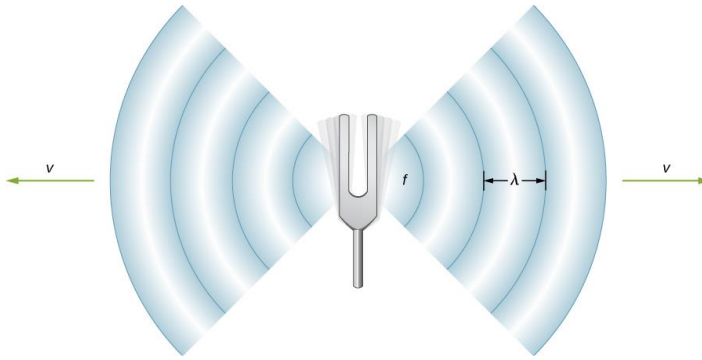


Figure 26 A sound wave emanates from a source, such as a tuning fork, vibrating at a frequency f . It propagates at speed v and has a wavelength λ .

Speed of Sound in Various Media

The speed of sound in a medium depends on how quickly vibrational energy can be transferred through the medium. For this reason, the derivation of the speed of sound in a medium depends on the medium and on the state of the medium. In general, the equation for the speed of a mechanical wave in a medium depends on the square root of the restoring force, or the elastic property, divided by the inertial property,

$$v = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}}$$

Also, sound waves satisfy the wave equation derived in Waves,

$$\frac{\partial^2 y(x, t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y(x, t)}{\partial t^2}$$

Recall from waves that the speed of a wave on a string is equal to $v = \sqrt{\frac{F_T}{\mu}}$ where the restoring force is the tension in the string F_T and the linear density μ is the inertial property. In a fluid, the speed of sound depends on the bulk modulus and the density,

$$v = \sqrt{\frac{B}{\rho}} \quad (4)$$

The speed of sound in a solid depends on the Young's modulus of the medium and the density,

$$v = \sqrt{\frac{Y}{\rho}} \quad (5)$$

In an ideal gas, the equation for the speed of sound is

$$v = \sqrt{\frac{\gamma R T_K}{M}} \quad (6)$$

where γ is the adiabatic index, $R=8.31$ J/mol. K is the gas constant, T_K is the absolute temperature in kelvins, and M is the molar mass. In general, the more rigid (or less compressible) the medium, the faster the speed of sound. This observation is analogous to

the fact that the frequency of simple harmonic motion is directly proportional to the stiffness of the oscillating object as measured by k , the spring constant. The greater the density of a medium, the slower the speed of sound. This observation is analogous to the fact that the frequency of a simple harmonic motion is inversely proportional to the mass of the oscillating object. The speed of sound in air is low, because air is easily compressible. Because liquids and solids are relatively rigid and very difficult to compress, the speed of sound in such media is generally greater than in gases.

Because the speed of sound depends on the density of the material, and the density depends on the

temperature, there is a relationship between the temperature in a given medium and the speed of sound in the medium. For air at sea level, the speed of sound is given by

$$v = 331 \frac{m}{s} \sqrt{1 + \frac{T_C}{273C}} = 331 \frac{m}{s} \sqrt{\frac{T_K}{273K}} \quad (7)$$

The speed of sound in gases is related to the average speed of particles in the gas, $v_{rms} = \sqrt{\frac{3k_B T}{m}}$ where k_B is the Boltzmann constant (1.38×10^{-23} J/K) and m is the mass of each (identical) particle in the gas. Note that v refers to the speed of the coherent propagation of a disturbance (the wave), whereas v_{rms} describes the speeds of particles in random directions. Thus, it is reasonable that the speed of sound in air and other gases should depend on the square root of temperature.

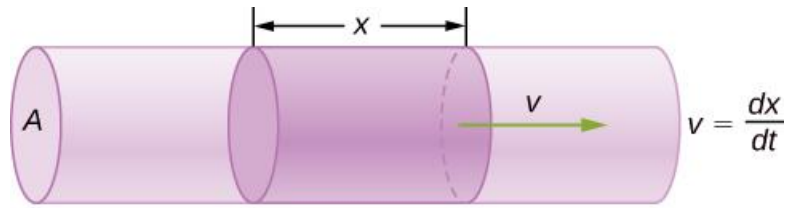
Derivation of the Speed of Sound in Air

the speed of sound in a medium depends on the medium and the state of the medium. The derivation of the equation for the speed of sound in air starts with the mass flow rate and continuity equation.

Consider fluid flow through a pipe with cross-sectional area A (Figure 27). The mass in a small volume of length x of the pipe is equal to the density times the volume, or $m = \rho V = \rho A x$. The mass flow rate is

$$\frac{dm}{dt} = \frac{d}{dt}(\rho V) = \frac{d}{dt}(\rho A x) = \rho A \frac{dx}{dt} = \rho A v$$

The continuity equation from Fluid Mechanics states that the mass flow rate into a volume has to equal the mass flow rate out of the volume, $\rho_{in} A_{in} v_{in} = \rho_{out} A_{out} v_{out}$



$$m = \rho V = \rho Ax$$

$$\frac{dm}{dt} = \rho A \frac{dx}{dt} = \rho Av$$

Figure 27 The mass of a fluid in a volume is equal to the density times the volume.

Now consider a sound wave moving through a parcel of air. A parcel of air is a small volume of air with imaginary boundaries (Figure 28). The density, temperature, and velocity on one side of the volume of the fluid are given ρ, T, v as and on the other side are $\rho + d\rho, T + dT, v + dv$.

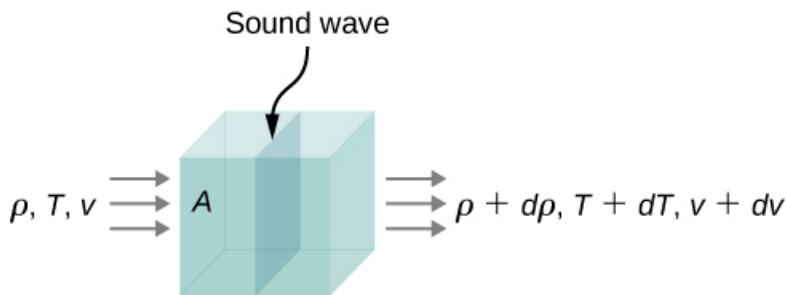


Figure 28 A sound wave moves through a volume of fluid.

The continuity equation states that the mass flow rate entering the volume is equal to the mass flow rate leaving the volume, so

$$\rho Av = (\rho + d\rho) A(v + dv)$$

This equation can be simplified, noting that the area cancels and considering that the multiplication of two infinitesimals is approximately equal to zero: $d\rho(dv) \approx 0$

$$\rho v = (\rho + d\rho)(v + dv)$$

$$\rho v = \rho v + \rho(dv) + (d\rho)v + (d\rho)(dv)$$

$$0 = \rho(dv) + (d\rho)v$$

$$\rho dv = -vd\rho.$$

The net force on the volume of fluid (Figure 29) equals the sum of the forces on the left face and the right face:

$$\begin{aligned}
 F_{\text{net}} &= p \, dy \, dz - (p + dp) \, dy \, dz \\
 &= p \, dy \, dz - p \, dy \, dz - dp \, dy \, dz \\
 &= -dp \, dy \, dz \\
 ma &= -dp \, dy \, dz.
 \end{aligned}$$

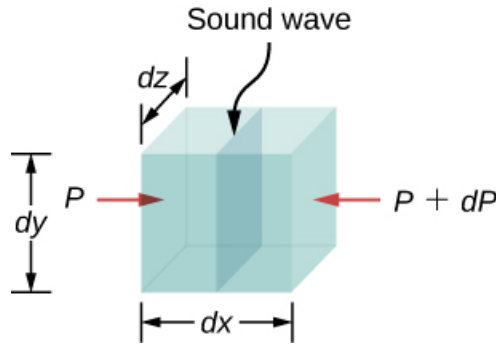


Figure 29 A sound wave moves through a volume of fluid.

The acceleration is the force divided by the mass and the mass is equal to the density times the volume,

$$m = \rho V = \rho \, dx \, dy \, dz. \text{ We have}$$

$$\begin{aligned}
 ma &= -dp \, dy \, dz \\
 a &= \frac{-dp \, dy \, dz}{m} = \frac{-dp \, dy \, dz}{\rho \, dx \, dy \, dz} = \frac{-dp}{(\rho \, dx)} \\
 \frac{dv}{dt} &= \frac{-dp}{(\rho \, dx)} \\
 dv &= \frac{-dp}{(\rho \, dx)} dt = \frac{-dp}{\rho} \frac{1}{v} \\
 \rho v \, dv &= -dp.
 \end{aligned}$$

From the continuity equation $\rho \, dv = -v \, d\rho$, we obtain

$$\begin{aligned}
 \rho v \, dv &= -dp \\
 (-v \, d\rho) v &= -dp \\
 v &= \sqrt{\frac{dp}{d\rho}}.
 \end{aligned}$$

Consider a sound wave moving through air. During the process of compression and expansion of the gas, no heat is added or removed from the system. A process where heat is not added or removed from the system is known as an adiabatic system. For an adiabatic process, pV^γ where p is the pressure, V is the volume, and γ is a constant that depends on the gas. For air, $\gamma = 1.40$. The density equals the number of moles times the molar mass divided by the volume, so the volume is equal to $V = \frac{nM}{\rho}$. The number of moles and the molar mass are constant and can be absorbed into the constant $p\left(\frac{1}{\rho}\right)^\gamma =$

constant Taking the natural logarithm of both sides yields $\ln p - \gamma \ln \rho = \text{constant}$
 Differentiating with respect to the density, the equation becomes:

$$\begin{aligned} \ln p - \gamma \ln \rho &= \text{constant} \\ \frac{d}{d\rho}(\ln p - \gamma \ln \rho) &= \frac{d}{d\rho}(\text{constant}) \\ \frac{1}{p} \frac{dp}{d\rho} - \frac{\gamma}{\rho} &= 0 \\ \frac{dp}{d\rho} &= \frac{\gamma p}{\rho}. \end{aligned}$$

If the air can be considered an ideal gas, we can use the ideal gas law:

$$\begin{aligned} pV &= nRT = \frac{m}{M} RT \\ p &= \frac{m}{V} \frac{RT}{M} = \rho \frac{RT}{M}. \end{aligned}$$

Here M is the molar mass of air:

$$\frac{dp}{d\rho} = \frac{\gamma p}{\rho} = \frac{\gamma \left(\rho \frac{RT}{M} \right)}{\rho} = \frac{\gamma RT}{M}.$$

Since the speed of sound is equal to $v = \sqrt{\frac{dp}{d\rho}}$, the speed is equal to :

$$v = \sqrt{\frac{\gamma RT}{M}}.$$

Note that the velocity is faster at higher temperatures and slower for heavier gases. For air, $\gamma = 1.4$, $M = 0.02897 \frac{\text{kg}}{\text{mol}}$, and $R = 8.31 \frac{\text{J}}{\text{mol}\cdot\text{K}}$. If the temperature is $T_C = 20^\circ\text{C}$ ($T = 293 \text{ K}$), the speed of sound is $v = 343 \text{ m/s}$.

The equation for the speed of sound in air $v = \sqrt{\frac{\gamma RT}{M}}$ can be simplified to give the equation for the speed of sound in air as a function of absolute temperature:

$$\begin{aligned} v &= \sqrt{\frac{\gamma RT}{M}} \\ &= \sqrt{\frac{\gamma RT}{M} \left(\frac{273 \text{ K}}{273 \text{ K}} \right)} = \sqrt{\frac{(273 \text{ K})\gamma R}{M}} \sqrt{\frac{T}{273 \text{ K}}} \\ &\approx 331 \frac{\text{m}}{\text{s}} \sqrt{\frac{T}{273 \text{ K}}}. \end{aligned}$$

One of the more important properties of sound is that its speed is nearly independent of the frequency. This independence is certainly true in open air for sounds in the audible range.

$$v = \lambda f$$

In a given medium under fixed conditions, v is constant, so there is a relationship between f and λ the higher the frequency, the smaller the wavelength.

Example// Calculate the wavelengths of sounds at the extremes of the audible range, 20 and 20,000 Hz, in 30 C air.

Solution//

1. Identify knowns. The value for v is given by

$$v = (331 \text{ m/s}) \sqrt{\frac{T}{273 \text{ K}}}.$$

2. Convert the temperature into kelvins and then enter the temperature into the equation

$$v = (331 \text{ m/s}) \sqrt{\frac{303 \text{ K}}{273 \text{ K}}} = 348.7 \text{ m/s}.$$

3. Solve the relationship between speed and wavelength for λ :

$$\lambda = \frac{v}{f}.$$

4. Enter the speed and the minimum frequency to give the maximum wavelength:

$$\lambda_{\text{max}} = \frac{348.7 \text{ m/s}}{20 \text{ Hz}} = 17 \text{ m}.$$

5. Enter the speed and the maximum frequency to give the minimum wavelength:

$$\lambda_{\text{min}} = \frac{348.7 \text{ m/s}}{20,000 \text{ Hz}} = 0.017 \text{ m} = 1.7 \text{ cm}.$$