Periodic & Oscillatory Motion:-

The motion in which repeats after a regular interval of time is called periodic motion.

1. The periodic motion in which there is existence of a restoring force and the body moves along the same path to and fro about a definite point called equilibrium position/mean position, is called oscillatory motion.

2. In all type of oscillatory motion one thing is common i.e each body (performing oscillatory motion) is subjected to a restoring force that increases with increase in displacement from mean position.

Types of oscillatory motion:-

It is of two types such as linear oscillation and circular oscillation.

Example of linear oscillation:-

1. Oscillation of mass spring system.
2. Oscillation of fluid column in a U-tube.
3. Oscillation of floating cylinder.
4. Oscillation of body dropped in a tunnel along earth diameter.
5. Oscillation of strings of musical instruments.

Example of circular oscillation:-

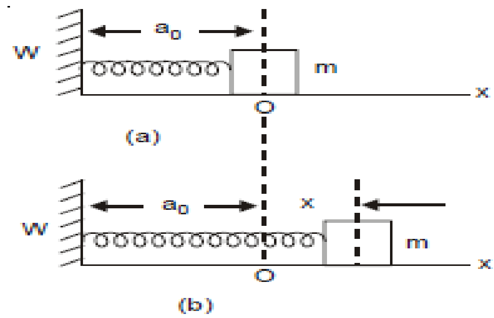
1. Oscillation of simple pendulum.
2. Oscillation of solid sphere in a cylinder (If solid sphere rolls without slipping).
3. Oscillation of a circular ring suspended on a nail.
4. Oscillation of balance wheel of a clock.
5. Rotation of the earth around the sun.

Simple Harmonic Motion

A very common type of periodic motion is called simple harmonic motion (SHM). A system that oscillates with SHM is called a simple harmonic oscillator.

Particle is said to be execute simple harmonic oscillation is the restoring force is directed towards the equilibrium position and its magnitude is directly proportional to the magnitude and displacement from the equilibrium position.

If the net force can be described by Hooke's law and there is no damping (slowing down due to friction or other nonconservative forces), then a simple harmonic oscillator oscillates with equal displacement on either side of the equilibrium position.as shown for an object on a spring in Figure 16.



Two important factors affect the period of a simple harmonic oscillator. the mass m and the force constant k are the only factors that affect the period and frequency of SHM. Note that the force constant is sometimes referred to as the spring constant.

If F is the restoring force on the oscillator when its displacement from the equilibrium position is x, then

$$F \alpha -x$$

Here, the negative sign implies that the direction of restoring force is opposite to that of displacement of the body i.e towards the equilibrium position.

$$F= -kx \qquad (1)$$

Where k= proportionality constant called force constant.

$$Ma = -kx \qquad \Rightarrow M\frac{d^2y}{dt^2} + kx = 0$$

$$\frac{d^2y}{dt^2} + \omega^2 x = 0 \qquad (2)$$

Here $\omega = \sqrt{\frac{k}{m}}$ is the angular frequency of the oscillation. The angular frequency depends only on the force constant and the mass, and not the amplitude. The angular frequency yields an equation for the period of the motion:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

The period also depends only on the mass and the force constant. The greater the mass, the longer the period.
The stiffer the spring, the shorter the period. The frequency is:

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Equation (2) is called general differential equation of SHM. By solving this differential equation:

$$x = \alpha e^{-i\omega t} + \beta e^{i\omega t} \qquad (3)$$

Where α, β are two constants which can be determined from the initial condition of a physical system.
Appling de-Moiver‟s theorem:

$$x = \alpha(cos\omega t + isin\omega t) + \beta(cos\omega t - isin\omega t)$$

$$x = (\alpha + \beta)cos\omega t + (\alpha - \beta)sin\omega t$$

$$x = Ccos\omega t + Dsin\omega t \qquad (4)$$

$$where\ C = \alpha + \beta \qquad ;D = \alpha - \beta$$

Let assume, C=A sinϕ , D=A cosϕ

Putting these values in equation (4)

$$x = Asin\phi cos\omega t + Acos\phi sin\omega t$$

$$x = Asin\phi cos\omega t + cos\phi sin\omega t$$

$$x(t) = A\sin(\omega t + \phi) \qquad (5)$$

$$where\ A = \sqrt{(C^2 + D^2)} \qquad \& \quad \phi = tan^{-1}(\frac{C}{D})$$

Similarly, the solution of differential equation can be given as

$$x(t) = A\cos(\omega t + \phi) \qquad (6)$$

Equation (5) and (6) represents the generalized equation for SHM where t is the time measured in seconds, is the angular frequency with units of inverse seconds, A is the amplitude measured in meters or centimeters, and ϕ is the phase shift measured in radians. It should be noted that because sine and cosine functions differ only by a phase shift, this motion could be modeled using either the cosine or sine function.

Velocity, Acceleration of a Simple harmonic oscillator

The velocity of the mass on a spring, oscillating in SHM, can be found by taking the derivative of the position equation:

$$v(t) = \frac{dx}{dt} = \frac{d}{dt}(A\cos(\omega t + \phi)) = -A\omega\sin(\omega t + \omega) = -v_{\max}\sin(\omega t + \phi).$$

Because the sine function oscillates between –1 and +1, the maximum velocity is the amplitude times the angular frequency, $v_{max} = A\omega$. The maximum velocity occurs at the equilibrium position x=0 when the mass is moving toward x=+A . The maximum velocity in the negative direction is attained at the equilibrium position x=0 when the mass is moving toward x=-A and is equal to -v_{max} .

The acceleration of the mass on the spring can be found by taking the time derivative of the velocity:

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}(-A\omega\sin(\omega t + \phi)) = -A\omega^2\cos(\omega t + \varphi) = -a_{\max}\cos(\omega t + \phi)$$

The maximum acceleration is$a_{max} = A\omega^2$. The maximum acceleration occurs at the position (x=-A), and

the acceleration at the position (x=-A) and is equal to $-a_{max}$.

In summary, the oscillatory motion of a block on a spring can be modeled with the following equations of

motion:

$$\begin{gathered} x(t) = A\cos(\omega t + \phi) \\ v(t) = -v_{\max}\sin(\omega t + \phi) \\ a(t) = -a_{\max}\cos(\omega t + \phi) \\ x_{\max} = A \\ v_{\max} = A\omega \\ a_{\max} = A\omega^2. \end{gathered}$$

The variation of displacement, velocity and acceleration with respect to time during SHM are as shown in the below figure 17.

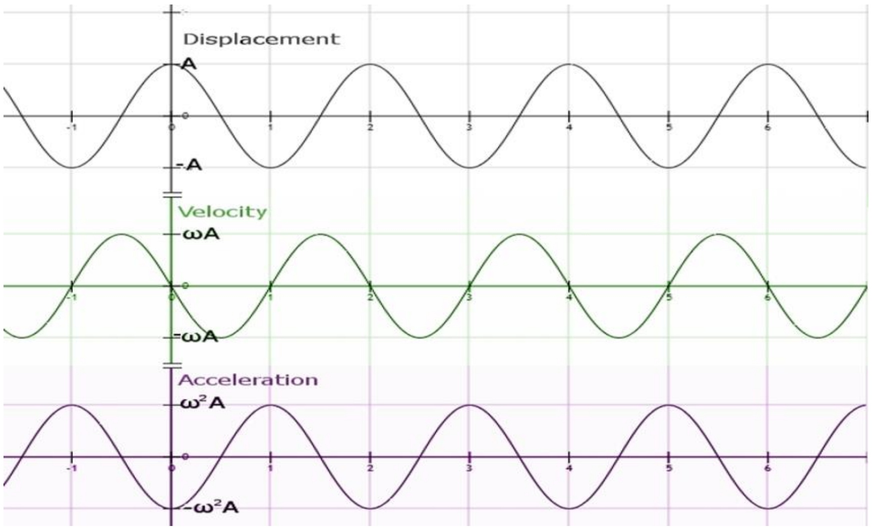


Figure .17

Example//A 2kg block is placed on a frictionless surface. A spring with a force constant of k=32 N/m is attached to the block, and the opposite end of the spring is attached to the wall. The spring can be compressed or extended. The equilibrium position is marked as x=0.

Work is done on the block, pulling it out to x=+0.02 m. The block is released from rest and oscillates between x=+0.02m and -0.02 m. The period of the motion is 1.57 s. Determine the equations of motion.

Solution//

The angular frequency can be found and used to find the maximum velocity and maximum acceleration:

$$\begin{aligned} \omega &= \frac{2\pi}{1.57\ \text{s}} = 4.00\ \text{s}^{-1}; \\ v_{\max} &= A\omega = 0.02\text{m}\left(4.00\ \text{s}^{-1}\right) = 0.08\ \text{m/s}; \\ a_{\max} &= A\omega^2 = 0.02\ \text{m}\left(4.00\ \text{s}^{-1}\right)^2 = 0.32\ \text{m/s}^2. \end{aligned}$$

All that is left is to fill in the equations of motion:

$$\begin{aligned} x(t) &= A\cos(\omega t + \phi) = (0.02\ \text{m})\cos\left(4.00\ \text{s}^{-1} t\right); \\ v(t) &= -v_{\max}\sin(\omega t + \phi) = (-0.08\ \text{m/s})\sin\left(4.00\ \text{s}^{-1} t\right); \\ a(t) &= -a_{\max}\cos(\omega t + \phi) = \left(-0.32\ \text{m/s}^2\right)\cos\left(4.00\ \text{s}^{-1} t\right). \end{aligned}$$