Energy in Simple Harmonic Motion

Consider the example of a block attached to a spring on a frictionless table, oscillating in SHM. The force of the spring is conservative, and we can define a potential energy for it. This potential energy is the energy stored in the spring when the spring is extended or compressed. In this case, the block oscillates in one dimension with the force of the spring acting parallel to the motion:

$$W = \int_{x_i}^{x_f} F_x dx = \int_{x_i}^{x_f} -kx dx = \left[-\frac{1}{2}kx^2\right]_{x_i}^{x_f} = -\left[\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2\right] = -\left[U_f - U_i\right] = -\Delta U.$$

When considering the energy stored in a spring, the equilibrium position, marked as is x=0 the position at which the energy stored in the spring is equal to zero. When the spring is stretched or compressed a distance x, the potential energy stored in the spring is:

$$U = \frac{1}{2}kx^2$$

In a simple harmonic oscillator, the energy oscillates between kinetic energy of the mass $K - \frac{1}{2}mv^2$ and potential energy $U = \frac{1}{2}kx^2$ stored in the spring. In the SHM of the mass and spring system, there are no dissipative forces, so the total energy is the sum of the potential energy and kinetic energy.
the energy oscillates between the kinetic energy of the block and the potential energy stored in the spring:

$$E_{\text{Total}} = U + K = \frac{1}{2}kx^2 + \frac{1}{2}mv^2.$$

The motion of the block on a spring in SHM is defined by the position $x(t) = A\cos(\omega t + \phi)$ with a velocity of $v(t) = -A\omega \sin(\omega t + \phi)$. Using these equations, the trigonometric identity $cos^2 + sin^2 = 1$ and,$\omega = \sqrt{\frac{k}{m}}$,we can find the total energy of the system:

$$\begin{aligned} E_{\text{Total}} &= \frac{1}{2}kA^2\cos^2(\omega t+\phi)+\frac{1}{2}mA^2\omega^2\sin^2(\omega t+\phi) \\ &= \frac{1}{2}kA^2\cos^2(\omega t+\phi)+\frac{1}{2}mA^2\left(\frac{k}{m}\right)\sin^2(\omega t+\phi) \\ &= \frac{1}{2}kA^2\cos^2(\omega t+\phi)+\frac{1}{2}kA^2\sin^2(\omega t+\phi) \\ &= \frac{1}{2}kA^2\left(\cos^2(\omega t+\phi)+\sin^2(\omega t+\phi)\right) \\ &= \frac{1}{2}kA^2. \end{aligned}$$

The total energy of the system is constant.

Pendulums

The Simple Pendulum

A simple pendulum is defined to have a point mass, also known as the pendulum bob, which is suspended from a string of length L with negligible mass (Figure 18). Here, the only forces acting on the bob are the force of gravity (i.e., the weight of the bob) and tension from the string. The mass of the string is assumed to be negligible as compared to the mass of the bob.

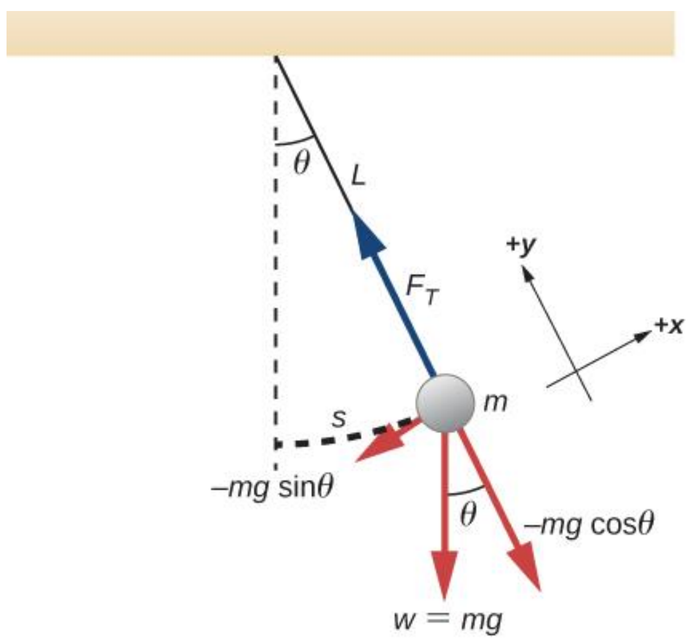

Figure 18: A simple pendulum

Consider the torque on the pendulum. The force providing the restoring torque is the component of the weight of the pendulum bob that acts along the arc length. The torque is the length of the string L times the component of the net force that is perpendicular to the radius of the arc. The minus sign indicates the torque acts in the opposite direction of the angular displacement:

$$\begin{aligned}
\tau &= -L\,(mg\sin\theta)\,; \\
I\alpha &= -L\,(mg\sin\theta)\,; \\
I\frac{d^2\theta}{dt^2} &= -L\,(mg\sin\theta)\,; \\
mL^2\frac{d^2\theta}{dt^2} &= -L\,(mg\sin\theta)\,; \\
\frac{d^2\theta}{dt^2} &= -\frac{g}{L}\sin\theta.
\end{aligned}$$

But note that for small angles (less than 15 degrees), sinθ and θ differ by less than 1%, so we can use the small angle approximation $sin\theta \approx \theta$.The angle describes the position of the pendulum. Using the small angle approximation gives an approximate solution for small angles,

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$$

Because this equation has the same form as the equation for SHM, the solution is easy to find. The angular
frequency is:

$$\omega = \sqrt{\frac{g}{L}}$$

and the period is: $T = 2\pi\sqrt{\frac{g}{L}}$

The period of a simple pendulum depends on its length and the acceleration due to gravity. The period is completely independent of other factors, such as mass and the maximum displacement.