The wave function ψ and its physical Significance

From wave particle duality, we know that a wave is associated with a particle. In quantum mechanics, the physical condition of a particle i.e., its state is described by a wave function which is a function of position and time so that:

$$\psi = \psi(r,t) \qquad (1)$$

The wave function though itself has no physical interpretation yet it contains all relevant information about the physical state of the particle and thus describes it completely.

- ψ In general is a complex quantity so that:

$$\psi^{*}\psi = |\psi(r,t)|^{2} \qquad (2)$$

is a physically meaningful quantity defines the probability density. It is a real positive quantity which satisfies,

$$\int_{-\infty}^{+\infty} |\psi|^{2} dr = 1 \qquad (3)$$

meaning, that particle exists somewhere at all times. The equation (3) is known as normalization condition. $\boxed{\text{or}}$:

$$\boxed{\int_{-\infty}^{+\infty} |\psi(x,y,z)|^2 dx\,dy\,dz = 1}$$

ψ is a normalized wave function.

Now, if

$$\int \psi_1^* \psi_1 dv = K \qquad (4)$$

if ψ_1 is not normalized wave function, i.e., $k \neq 1$. one can obtain a normalized wave function by multiplying (ψ_1) by a certain constant be (R), that :

$$\psi = R\psi_1 \qquad (5)$$

Sub. eq. (5) into relation (3), one has

$$\int |\psi|^2 dv = \int \psi^* \psi\, dv = 1 \Rightarrow |R|^2 \int \psi_1^* \psi_1 dv = 1$$

$$\because \int \psi_1^* \psi_1 dv = K$$

$$\Rightarrow |R|^2 \cdot K = 1 \Rightarrow R = \frac{1}{\sqrt{K}}$$

$\boxed{\text{or}}$ $R = \dfrac{1}{\left(\int |\psi_1|^2 dv\right)^{1/2}}$

In general, the orthonormality condition is given by,

$$\int \psi_i^* \psi_j \, dv = \delta_{ij} \begin{cases} 0 & ; \ i \neq j \\ 1 & ; \ i = j \end{cases}$$

δ_{ij} is known as Delta kronicker.

A wave function would be adimmisble as a mathematical expression of state of a particle if and only if it satifies the following conditions:

1. ψ is continuous and single valued every where.
2. $\frac{\partial \psi}{\partial x}$, $\frac{\partial \psi}{\partial y}$ and $\frac{\partial \psi}{\partial z}$ are continuous and single value every where.
3. It must be normalizable. For this ψ must go to 0 as $x \rightarrow \pm\infty$, $y \rightarrow \pm\infty$, $z \rightarrow \pm\infty$ in order that $\int |\psi(r,t)|^2 dr$ over all space be a finite constant.

Linear operators

The result obtained after performing a given mathematical operation upon a wave function ψ is symbolized as,

$$\phi = A_{op}\psi$$

or $$\boxed{\phi = \hat{A}\psi} \longrightarrow (1)$$

for an example,

$$\hat{A} = \frac{d}{dx}$$

$$\psi = x^3$$

$$\Rightarrow \frac{d}{dx}(x^3) = 3x^2$$

$\hat{A}$ is called a linear operator if it satisfies,

$$\begin{cases} 1.\ \hat{A}(\psi_1+\psi_2) = \hat{A}\psi_1 + \hat{A}\psi_2 \\ 2.\ \hat{A}(c\psi_1) = c\hat{A}\psi_1 \end{cases}$$

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where ψ_1 and ψ_2 are two given functions and c is a constant (may be a complex number as well).

- Examples of linear operators $\left\{ x_i,\ \frac{\partial}{\partial x_i},\ \nabla^2,\ \frac{\partial}{\partial t} \right\}$