Commutation Relations علاقات التبادل

Let A, B be operators, so the commutator is defined by:

$$[A, B] = AB - BA \quad \text{——— (1)}$$

for example,

$$\left[x_i, \frac{\partial}{\partial x_j}\right] = \left(x_i \frac{\partial}{\partial x_j} - \frac{\partial}{\partial x_j} x_i\right)\psi$$

$$= \cancel{x_i \frac{\partial \psi}{\partial x_j}} - \cancel{x_i \frac{\partial \psi}{\partial x_j}} - \frac{\partial x_i}{\partial x_j}\psi$$

$$= \underline{-\delta_{ij}\psi}$$

$$\Rightarrow \left[x_i, \frac{\partial}{\partial x_j}\right] = -\delta_{ij} \quad ; \quad \left[\frac{\partial}{\partial x_j}, x_i\right] = \underline{+\delta_{ij}}$$

Here δ_{ij} is the Kronecker delta.

Example:

$$\left[\frac{\partial}{\partial x_i}, \frac{\partial}{\partial x_j}\right] = 0$$ (order of differentiation Commute)

تبادل المؤثرات

Example: $[x_i, x_j] = 0$ (Real numbers also Commute)

Example: An operator O_1 is defined by

$O_1 \psi(x) = x \frac{d}{dx} \psi(x)$. Check for its linearity?

تأكد من أن المؤثر O_1

مؤثر خطي

Sol: An operator O_1 is said to be linear if:

$$O_1\{c_1 f_1(x) + c_2 f_2(x)\} = x\frac{d}{dx}\{c_1 f_1(x) + c_2 f_2(x)\}.$$

$$= x\left\{c_1\frac{df_1}{dx} + c_2\frac{df_2}{dx}\right\}$$

$$= c_1 x\frac{df_1}{dx} + c_2 x\frac{df_2}{dx}$$

Again, $c_1 O_1 f_1(x) + c_2 O_1 f_2(x)$

$$= c_1 x\frac{df_1}{dx} + c_2 x\frac{df_2}{dx}$$

Hence operator O_1 is linear.

"H.W" تمارين

Example / An operator O_2 is defined by

Check whether it is a linear operator or not. $O_2\psi(x) = x^3\psi(x)$

Sol. $O_2\{c_1 f_1(x) + c_2 f_2(x)\} = x^3\{c_1 f_1(x) + c_2 f_2(x)\}$

$$= c_1 x^3 f_1(x) + c_2 x^3 f_2(x)$$

$$= c_1 O_2 f_1(x) + c_2 O_2 f_2(x)$$

Hence operator O_2 is linear.

Remark: If there are $\hat{A}, \hat{B}, \hat{C}$ are linear operators, one has: قواعد

1- $[\hat{A},\hat{B}] + [\hat{B},\hat{A}] = 0$

2. $[\hat{A}, A] = 0$

3- $[\hat{A}\hat{B}, \hat{C}] = A[\hat{B}, \hat{C}] + [\hat{A}, \hat{C}]\hat{B}$

4. $[\hat{A}+\hat{B}, \hat{C}] = [A, C] + [B, C]$

5. $[A, \hat{B}\hat{C}] = [\hat{A}, \hat{B}]\hat{C} + \hat{B}[\hat{A}, \hat{C}]$

6. $[\hat{A}, b] = 0$, b is a constant.

امثلة على تبادل المؤثرات

Examples:

$$\begin{cases} [\hat{x}, \hat{P}_y] = 0 & \text{"Commute"} \\ [\hat{x}, \hat{y}] = 0 & \\ [\hat{P}_x, \hat{P}_y] = 0 & ; \ [\hat{P}_x, \hat{T}_K] = 0 \\ & \boxed{\text{or}}\ [H, P_x] = 0 \\ [\hat{x}, \hat{P}_x] \neq 0 & \text{"not commutative"} \end{cases}$$

كيفية صياغة المؤثرات

- Formulation of operators in Q. M.

let be $\Psi(x,t)$ a wave function describes the state of a system, that:

$$\Psi(x,t) = A e^{i(kx - \omega t)}$$

so $E = \hbar\omega$, $p = \hbar k \Rightarrow \Psi(x,t) = A e^{i(P_x x - E t)/\hbar}$