Geometric Optics

[اكتب العنوان الفرعي للمستند]

Lecturer. Dr. Thill Akeel Kadhum Almusawi

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Optics III

Refraction of light

Refraction of light is one of the most commonly observed phenomena. **Refraction** of light is defined as the change in direction or the bending of a wave passing from one medium to another due to the change in speed of the wave. Some natural phenomena occurring in nature where refraction of light takes place are the twinkling of stars, the formation of mirages and rainbows, and many more. The major cause of refraction to occur is the change in the speed of waves in different mediums, which is different due to the difference between the densities of the mediums.

Other waves like sound waves and water waves also experience refraction. Refraction makes it possible for us to have optical instruments such as magnifying glasses, lenses and prisms. It is also because of the refraction of light that we are able to focus light on our retina.

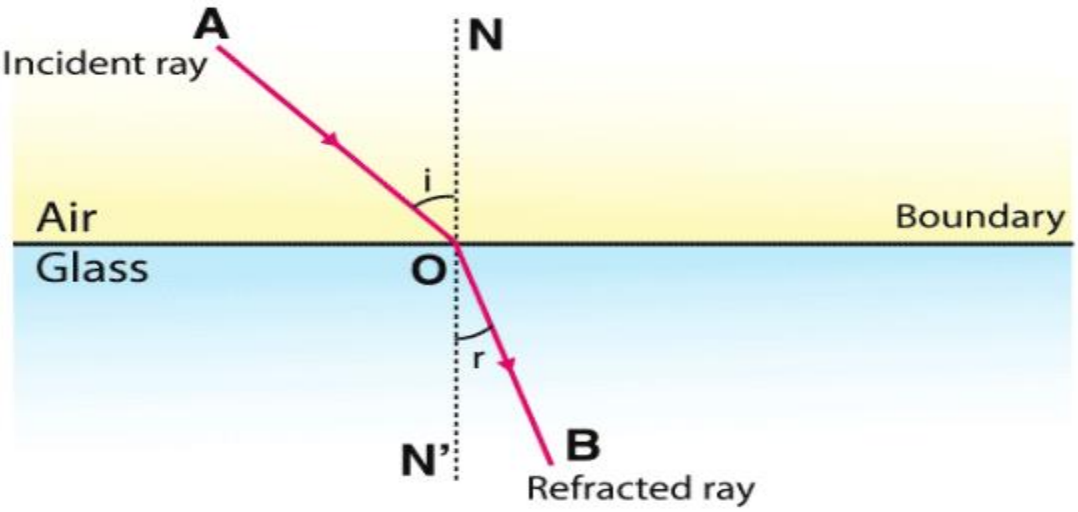


Fig. (23): Refraction of light.

The definitions of important terms used to study Refraction:

Normal: The line of the surface at which an optical phenomenon occurs is called the normal. It is shown by a dotted line drawn perpendicular to the surface of the refracting material, in a ray diagram.

Incident ray: The light rays that strike the refracting surface, at the separation of two media are called the incident ray.

Refracted ray: The light rays that bend after passing into another medium are called the refracted ray.
Angle of incidence: This is the angle between the incident ray and the normal. It is represented by (θ_i) and it is also called an incident angle.
Angle of Refraction: This is the angle between refracted ray and the normal. It is represented by (θ_r) and it is also called a refracted angle.

Refractive Index

The refractive index enables us to know how fast light travels through the material medium.

Refractive index is a dimensionless quantity. For a given material or medium, the refractive index is considered the ratio between the speed of light in a vacuum (c) to the speed of light in the medium (v) on which it goes. The refractive index for a medium is represented by small (n), and it is given by the following formula:

$$n = \frac{c}{v} \qquad (1-20)$$

Laws of Refraction of Light

The refraction of light traveling through different mediums follows some laws. There are two laws of refraction as stated below which at the sight of refraction, the light follows, and we see the refracted image of the object.

1- The reflected, incident, and the normal at the point of incidence all will tend to lie in the same plane.
2- The ratio of the sine of the angle of the incidence and refraction is constant which is termed Snell's law.

$$\frac{\sin(\theta_i)}{\sin(\theta_r)} = \text{constant (n)} \qquad (1-21)$$

where (θ_i) is the angle of incidence, (θ_r) is the angle of refraction, the constant value depends on the refractive indexes of the two mediums.

Snell's Law

Snell's law provides the degree of refraction that occurs through a relationship between the incident angle, refracted angles, and the refractive indices of a given pair of media.

According to Snell's law, the ratio of the sine of the incident angle to the sine of the refracted angle is a constant, for any light of a given color or for any given pair of media. The constant value is called the refractive index of the second medium with respect to the first.

Snell's law is given by the relation:

$$\frac{\sin(\theta_i)}{\sin(\theta_r)} = \frac{n_2}{n_1} \qquad \textbf{Snell's law} \qquad (1-22)$$

where (n) is the refractive index and (n_1) and (n_2) are the refractive indices of medium (1) and (2) respectively.

Fermat principle (Law of refraction)

Fermat's Principle: Light follows the path of least time. Snell's Law can be derived from this by setting the derivative of the time equal zero. We make use of the index of refraction, defined as (n=c/v).

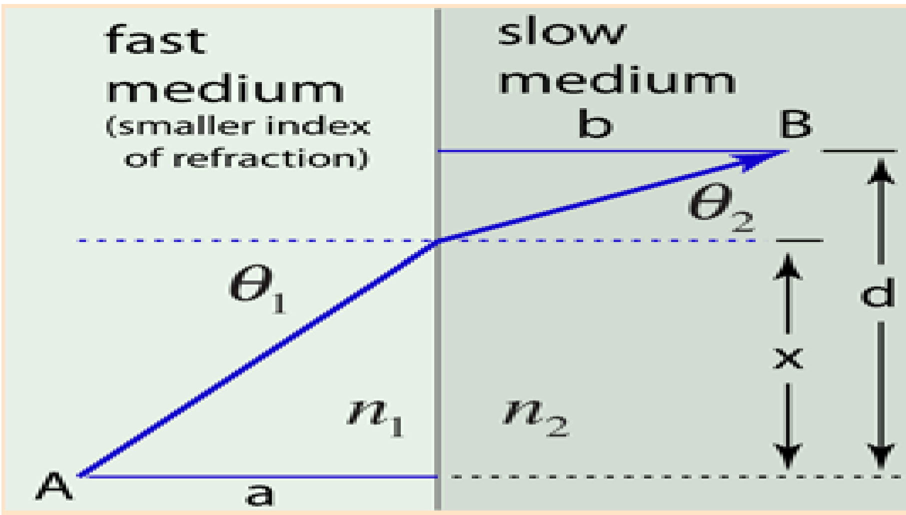


Fig. (24): Fermat principle for law of refraction.

The path length from A to B is:

$$t = \frac{\sqrt{a^2 + x^2}}{\nu} + \frac{\sqrt{b^2 + (d-x)^2}}{\nu'}$$

$$\frac{dt}{dx} = \frac{x}{\nu\sqrt{a^2 + x^2}} - \frac{(d-x)}{\nu'\sqrt{b^2 + (d-x)^2}} = 0$$

$$0 = \frac{\sin\theta_1}{\nu} - \frac{\sin\theta_2}{\nu'}$$

This reduces to:

$$\frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1} \qquad \textbf{Snell's Law} \qquad (1-23)$$

or:

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2) \qquad \textbf{Snell's Law} \qquad (1-24)$$

Refraction of Light by Lenses

Lens is piece of glass or other transparent substance that is used to form an image of an object by focusing rays of light from the object, usually circular in shape, with two polished surfaces, either or both of which is curved and may be either convex (bulging) or concave (depressed). The curves are always spherical; i.e., the radius of curvature is constant.

Single lenses are used in eyeglasses, contact lenses, pocket magnifiers, projection condensers, signal lights, viewfinders, and on simple box cameras. More often a number of lenses made of different materials are combined together as a compound lens in a tube to permit the correction of aberrations. Compound lenses are used in such instruments as cameras, microscopes, and telescopes.

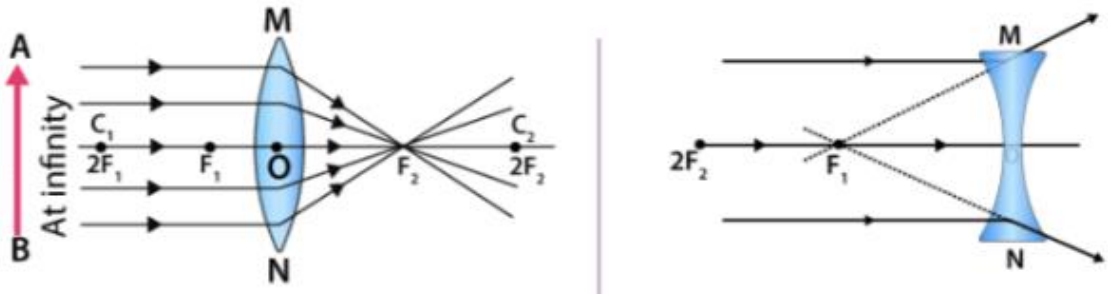


Fig.(25): The lenses.

Terminology of Spherical Mirrors

A few important terms when dealing with the concept of refraction by spherical lenses:

1- Centre of curvature (C): It is the centre of the hollow sphere of glass, from which the lens is derived.

2- Principal axis: It is the imaginary line joining the centres of curvatures of both spheres.

3- Principal focus (F): It is a point on the principal axis, where light rays parallel to the principal axis converge or appear to diverge. A lens has two principal foci represented by (F_1) and (F_2).

4- Optical center (O): It is a geometric centre of the curved lens or a point within the lens where the diameter of the lens and the principal axis intersect.

5- Focal length (f): It is the distance between the principal focus (F) and the optical centre.

6- Radius of curvature (R): it is the distance of the optical center from (C_1) or (C_2).

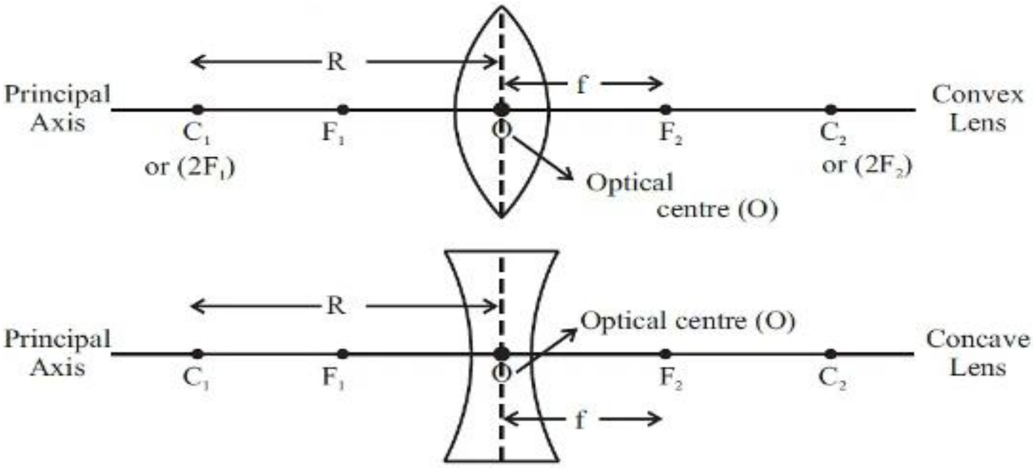


Fig.(27): Parts of spherical lenses.

Image Formation by Convex Lens

The image formation by Convex Lens through refraction is as follows:

1- When the object is placed at infinity: a real image is formed at the focus. The image is highly diminished and is point-sized.

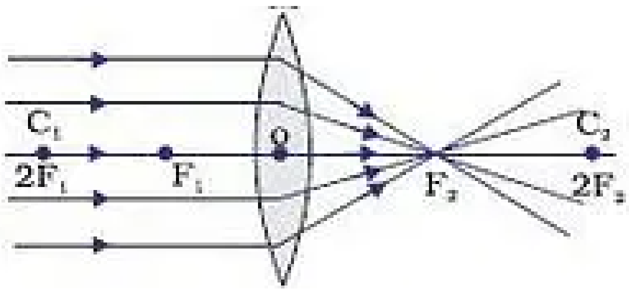


Fig. (28): object at infinity.

2- When the object is placed beyond the center of curvature: the image is formed between the center of curvature and focus. The image will be a real image and will be diminished in size.

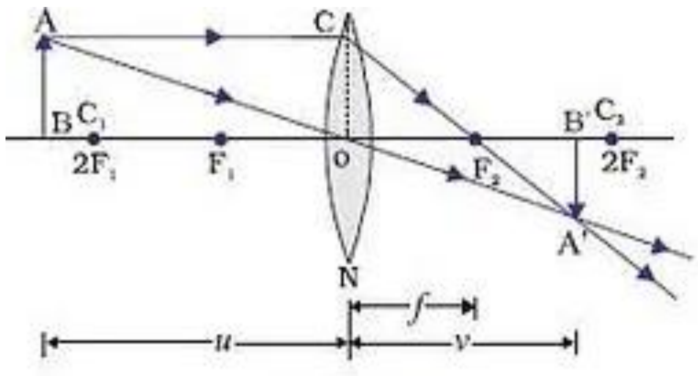


Fig.(29): object at beyond (2f).

3- When the object is placed at the center of curvature: a real image is formed at the other center of curvature which is of the same size as the object.

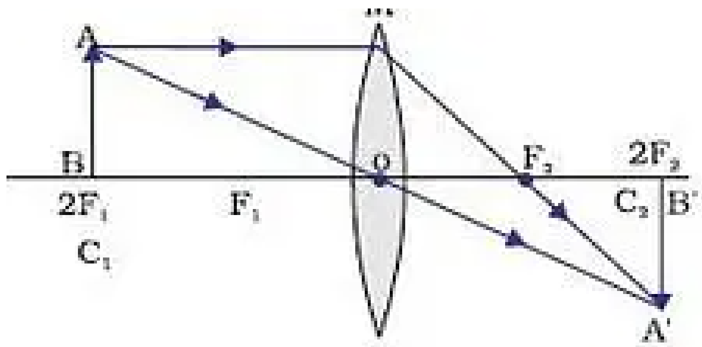


Fig.(30): Object at (2f).

4- When the object is placed in between the center of curvature and focus: a real image is formed behind the center of curvature which is larger than that of the object.

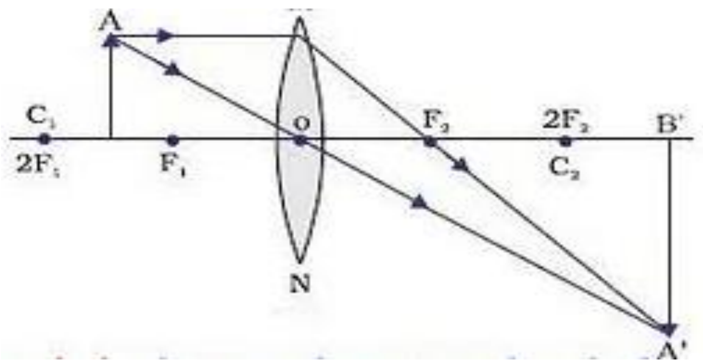


Fig.(31): Object between (f) and (2f).

5- When the object is placed at the focus: a real image is formed at infinity with a much larger size than that of the object.

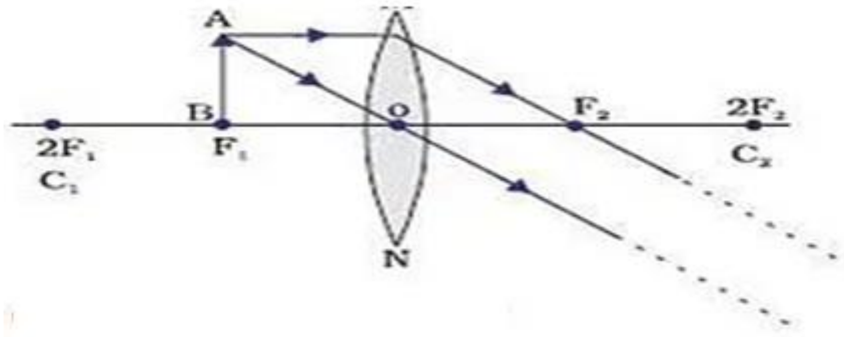


Fig.(32): Object at (f).

6- When the object is placed in between the focus and the optical center: a virtual image is formed with a larger size than the object.

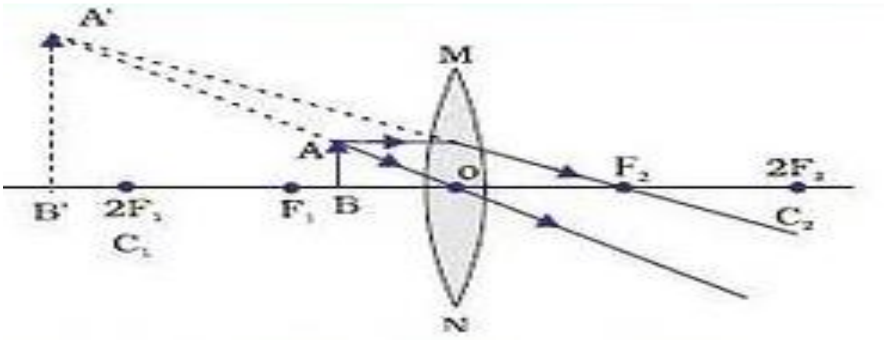


Fig.(33): Object between (F) and pole.

Refraction by Concave Lens

The image formation by concave lenses is as follows:

1- When the object is placed at infinity: a virtual image is formed at the focus which is highly diminished and point-sized.

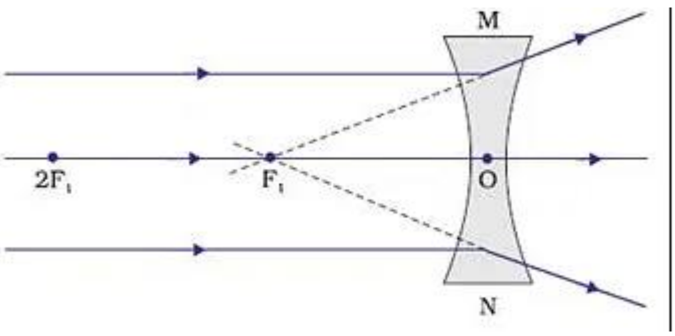


Fig.(34): Object at infinity.

2- When the object is placed at any finite distance: a virtual image is formed between the optical center and the focus with a much smaller size than that of the object.

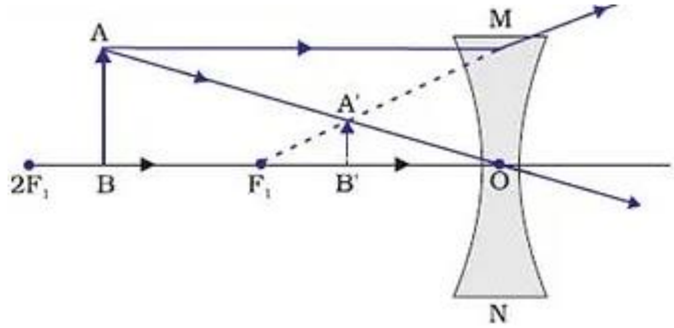


Fig.(35): Object between infinity and (O) of the lens.

Lens Maker Formula Derivation

If we have the thin lens shown in the image above with (2) refracting surfaces having the radii of curvatures (R1) and (R2) respectively, and let the refractive indices of the surrounding medium and the lens material be (n1) and (n2) respectively.

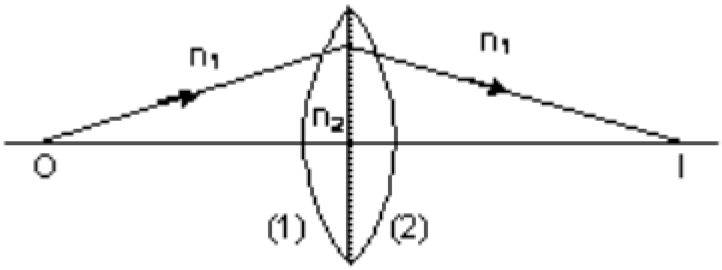


Fig.(36): Convex lens.

For the first surface:

$$\frac{n_1}{u} - \frac{n_2}{v} = \frac{n_1 - n_2}{R_1} \qquad (1-30)$$

For the second surface:

$$\frac{n_2}{u} - \frac{n_1}{v} = \frac{n_1 - n_2}{R_2} \qquad (1-31)$$

By adding equation (1-30) and (1-31), we get:

$$\frac{n_1}{u} - \frac{n_2}{v} = (n_2 - n_1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \qquad (1-32)$$

$$\frac{1}{u} - \frac{1}{v} = \left(\frac{n_2}{n_1} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \qquad (1-33)$$

But also:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

When $(u = \infty)$ and $(v = f)$

$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \qquad (1-34)$$

Therefore we can say that:

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \qquad (1-35)$$

where (n) is the refractive index of the material.

Power of a Lens (P)

The ability of a lens to bend the light falling on it is called the power of a lens. Since the lens of shorter focal length will bend the light rays more will have more power. A convex lens converges the light rays towards the principal axis whereas a concave lens diverges the light rays away from the principal axis.

The power of a lens is defined as the inverse of its focal length (f) in meters (m), and the unit of power of a lens known as diopter (D).

$$P = \frac{1}{f\,(m)} \quad (D) \qquad (1-36)$$

The focal length (f) of a converging lens is considered positive and that of a diverging lens is considered negative. Thus, the power of a converging lens is positive and that of the diverging lens is negative.

The power of any number of lenses in contact is equal to the algebraic sum of the power of two individual lenses:

$$P = P_1 + P_2 + P_3 + \cdots \qquad (1-37)$$

Examples:

1) A concave lens has focal length of (15cm). At what distance should the object from the lens be placed so that it forms an image at (10cm) from the lens? Also, find the magnification produced by the lens, and find power of this lens.

Solution:

A concave lens always form a virtual, erect image on the same side of the object.

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \Rightarrow \frac{1}{u}+\frac{1}{-10}=\frac{1}{-15} \Rightarrow u=-30\ \mathrm{cm}$$

$$m=\frac{v}{u} \quad \Rightarrow m=\frac{-10}{-30}=0.33$$

The positive sign shows that the image is erect and virtual. The image is one –third of the size of the object.

$$P=\frac{1}{f}=\frac{1}{-15}=6.667\ \mathrm{D}$$

2) A (2 cm) tall object is placed perpendicular to the principal axis of a convex lens of focal length (10 cm). The distance of the object from the lens is (15 cm). Find the nature, position and size of the image. Also find its magnification and power.

Solution:

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \Rightarrow \frac{1}{-15}+\frac{1}{v}=\frac{1}{10} \Rightarrow v=30\ \mathrm{cm}$$

The image is real and inverted.

$$m=\frac{h_i}{h_o}=-\frac{v}{u} \quad \Rightarrow h_i=-u\left(\frac{h_o}{v}\right)=-4\ \mathrm{cm}$$

$$m=\frac{30\ \mathrm{cm}}{-15\ \mathrm{cm}}=-2$$

The negative sign of (m) and (h_i) show that the image is inverted and real. The image is two times enlarged.

$$P=\frac{1}{f}=\frac{1}{0.1}=10\ \mathrm{D}$$

Compound lenses formula

Compound lens are the type of lenses that features two thin lenses that are mounted on a common axis usually closes to each other or often cemented together.

First: Two thin lenses in contact.

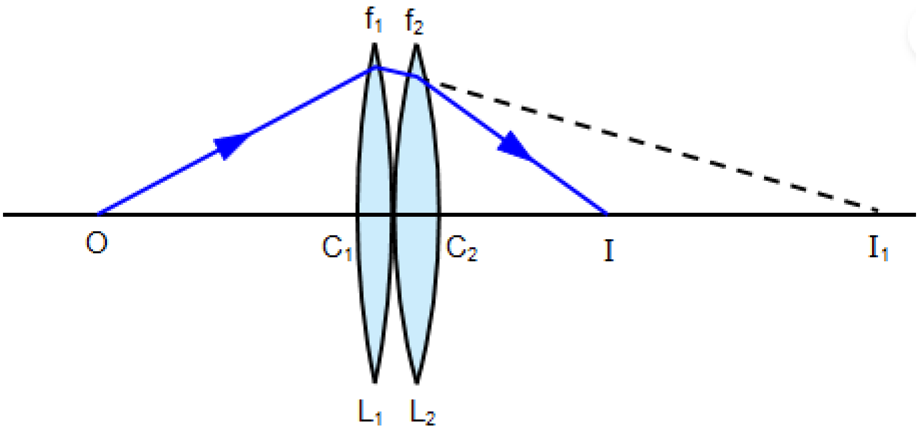


Fig. (37): Compound lens in contact.

In figure (37), let the focal lengths of the two lenses be (f1) and (f2). $(u_1 = OC_1)$ and $(v_1 = I_1C_1)$.

$(u_2 = -C_1I_1)$ which is approximately equal to $(u_2 = C_2I_1)$ and $(v_2 = C_2I)$ which is approximately equal to (C_1I). Therefore:

$$\frac{1}{u_1} + \frac{1}{v_1} = \frac{1}{f_1} \qquad \Rightarrow \qquad \frac{1}{OC_1} + \frac{1}{C_1I_1} = \frac{1}{f_1}$$

and

$$\frac{1}{u_2} + \frac{1}{v_2} = \frac{1}{f_2} \qquad \Rightarrow \qquad \frac{1}{-C_1I_1} + \frac{1}{C_1I} = \frac{1}{f_2}$$

Therefore:

$$\frac{1}{OC_1} + \frac{1}{C_1I} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{F}$$

Combined focal length (F) of two thin lenses in contact is given by:

$$F = \frac{f_1f_2}{[f_1 + f_2]} \qquad \textbf{Combined focal length} \qquad (1-38)$$

Second: **Two thin lenses separated by a distance (d).**

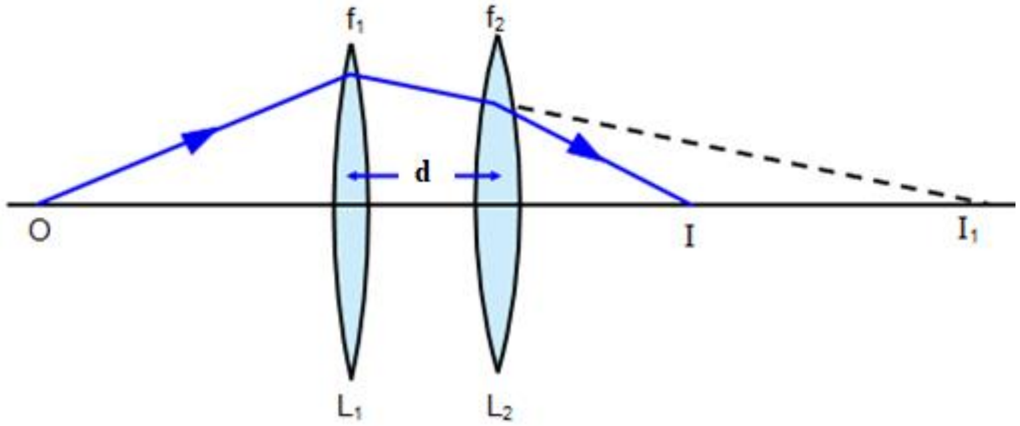


Fig. (38): Compound lens separated by a distance (d).

The combined focal length for two thin lenses separated by a distance (d) as figure (38) is given by the equation:

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \qquad \textbf{Combined focal length} \qquad (1-39)$$

where (d) is the thickness of the lens (the distance along the lens axis between the two surface vertices).

Homework:

1) A biconvex lens is made of glass with refractive index (1.52) having the radii of curvature of (20 cm) respectively. Determine the focal length of the lens in:

a) water with refractive index is (1.33).

b) carbon disulfide with refractive index is (1.63).

2) An object is placed (90 cm) from glass lens (n = 1.56) with one concave surface of radius (22 cm) and one convex surface of radius (18.5 cm). Determine:

a) the image position. b) the linear magnification.

3) A converging lens with of focal length of (90 cm) forms an image of a (3.2 cm) tall real object that is to the left of the lens. The image is (4.5 cm) tall and inverted. Find:

a) the object position from the lens.

b) the image position from the lens. Is the image real or virtual?

4) A bi-convex lens of focal length (10 cm) is fixed to a plano- concave lens of focal length (20 cm) made of glass of the same refractive index. What is the focal length of the combination?

Some of applications of refraction

1) Rainbow

2) Mirage

3) Totally reflecting prism

Prism: A prism is a solid figure whose bases have the same size and shape and are parallel to one another, and each of its sides is a parallelogram.

A total reflecting prism is defined as a prism with an angle of (90^0) between its two refracting surfaces and the other two angles each equal to (45^0). When light is incident normally on any of the prism's faces, it suffers total internal reflection within the prism.

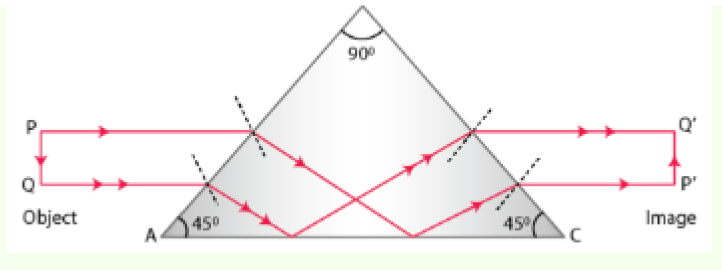


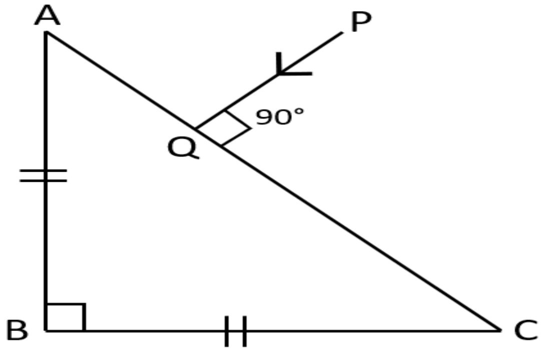
Fig. (39): Totally reflecting prism.

Note:

The condition necessary for total internal reflection is that the angle of incident should be more than the critical angle of prism and value of critical angle of glass is (42^0).

Homework:

1) In the figure, a ray of light (PQ) is incident normally on the hypotenuse of an isosceles right angled prism (ABC).



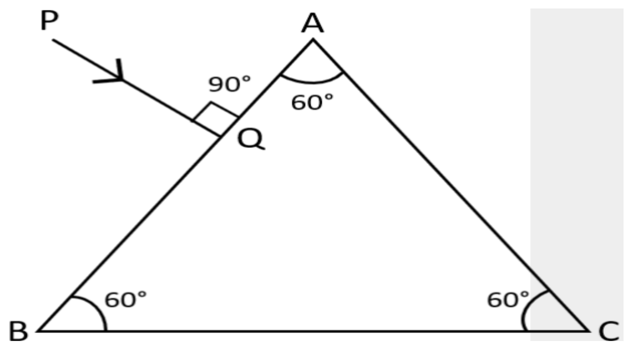
(a) Complete the path of the ray (PQ) till it emerges from the prism. Mark in the diagram the angle wherever necessary.

(b) What is the angle of deviation of the ray (PQ)?

(c) Name a device in which this action is used.

2) Draw a diagram of a right angled isosceles prism which is used to make an inverted image erect.

3) In the figure given below, a ray of light (PQ) is incident normally on the face (AB) of an equilateral glass prism. Complete the ray diagram showing its emergence into air after passing through the prism. Take critical angle for glass equals (42°).



a) Write the angles of incidence at the faces (AB) and (AC) of the prism.

b) Name the phenomenon which the ray of light suffers at the face (AB), (AC) and (BC) of the prism.

4) Sparkling of diamond

Total internal reflection, coupled with a large index of refraction, explains why diamonds sparkle more than other materials. The critical angle for a diamond-to-air surface is only (24.4°), so when light enters a diamond; it has trouble getting back out, as in figure (40).

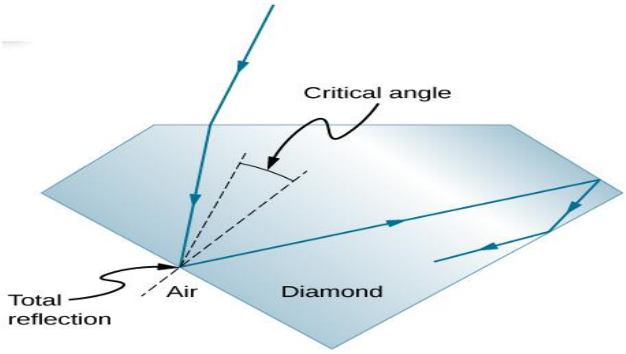


Fig. (40): Sparkling of Diamond.

Although light freely enters the diamond, it can exit only if it makes an angle less than (24.4°). Good diamonds are very clear, so that the light makes many internal reflections and is concentrated before exiting—hence the bright sparkle .

The colors you see emerging from a clear diamond are not due to the diamond’s color, which is usually nearly colorless, but result from dispersion.