Einstein theory gives the difference in the heat capacity values in higher and low temperature, but the specific heat values which is calculated by this theory at low temperature is more less than the experimental values. Therefore the success of Einstein theory is not completely. Where the reason is found after that because of the theory assumption that all atoms vibrate in the same angular frequency or in one angular frequency.

1-3 Phonons

The vibration modes in the solid materials was explained as a waves of indistinct particles group which is called phonons, the expression of phonon is similar to the expression of photon where the photon is the optical energy quantum, while the phonon is the lattice vibration quantum (acoustical energy quantum). It characteristics is unchanged, his static mass (m_0) equal to zero, his energy is equal to $\hbar\omega$ and the transfers momentum is ($p=\hbar k$). The acoustical waves known as a flood of phonons which carry the energy and the momentum of the wave and the velocity of the phonon equal to the acoustical velocity in the solid materials. The phonons are indistinct particles therefore it obeys to Bose-Einstein statistics where the statistical laws used to find or calculate the number of particles which is probable be in an energy like E_1 , E_2 , E_3, ………… etc.

The average number of the phonons in the thermal equilibrium can be calculate by the following equation:

$$<n>=\frac{1}{e^{\hbar\omega/k_BT}-1} \qquad \text{[Bose-Einstein statistics]}$$

1-4 The Density of states:

The most important which distinct continuous elastic media of solid material with a certain volume consist of separated atoms from each other is the unlimited number of probable vibration modes of that media. If this solid material is influence by an external effect, it is atoms will be displaced from it is equilibrium positions and formed limited number of vibration modes.

The density of states defined as the number of vibration modes in a certain range of angular frequency, form (ω) to ($\omega + d\omega$) and equal to $D(\omega)d\omega$.

1-4-1 The Density of states in one dimension:

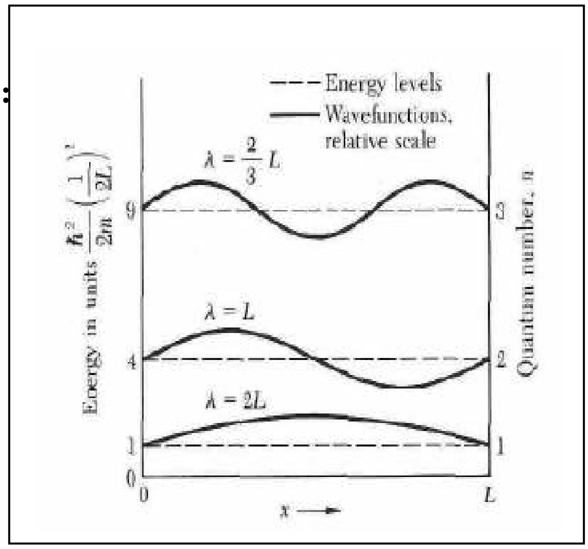
Consider the boundary value problem for vibration of one dimensional line of length (L) carrying N particle at separation (a), we suppose that the particles at the ends of line are held fixed. Along the distance (L) there is one or two or three ….etc. of half wave length ($\lambda/2$), that is mean the probable vibration modes is depend on the wave vector (k).

$$L = \frac{1}{2}\lambda \quad , \quad \lambda \quad , \quad \frac{3}{2}\lambda \quad , \quad ......, \quad n(\frac{\lambda}{2}) \qquad (9)$$

$$k = \frac{\pi}{L} \quad , \quad \frac{2\pi}{L} \quad , \quad \frac{3\pi}{L} \quad , \quad ...., \quad \frac{n\pi}{L} \qquad (10)$$

In order to get equation above. Let us use k-space. So, we have the following wave equation function :

$$u = A \exp\, i(kx - \omega t) \qquad (11)$$



As well as the following periodic boundary conditions:

$$u(x) = u(x + L) \qquad (12)$$

So, using the boundary conditions:

$$Ae^{i(kx-\omega t)} = Ae^{i(kx-\omega t)}.e^{ikL}$$

That is: $\qquad e^{ikL} = 1$

Hence, we see that k must be quantized thus:

$$k = \frac{2\pi n}{L}, \quad n = \pm 1 \quad , \quad \pm 2 \quad , \quad \pm 3, ..., \frac{N}{2} \qquad (13)$$

$$k = \frac{2\pi}{L}, \frac{4\pi}{L}, \frac{6\pi}{L}, \ldots\ldots, \quad \frac{N}{L}$$

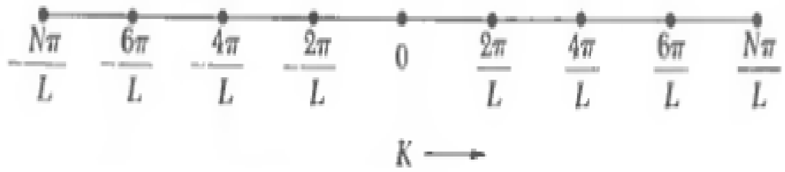


Figure 5 Allowed values of wavevector K for periodic boundary conditions applied to a linear lattice of periodicity $N = 8$ atoms on a line of length L. The $K = 0$ solution is the uniform mode.

The number of vibration modes is

$$N = \frac{k}{(2\pi / L)} \tag{14}$$

The Density state is

$$\boxed{D(\omega) = \frac{dN}{d\omega} = \frac{dN}{dk}\frac{dk}{d\omega} = \frac{L}{2\pi}\frac{dk}{d\omega}} \tag{15}$$

But $\omega = v_0 k$

$$\boxed{D(\omega) = \frac{L}{2\pi}\frac{1}{v_0}}$$