Examples 1: If $z_1 = 3+2i$, and $z_2 = 1-2i$. Evaluate $\dfrac{2z_2 - z_1 + 5 - i}{2z_1 - z_2 + 3 - i}$

and express the result in the form of $x + iy$

Solution

$$\frac{2z_2 - z_1 + 5 - i}{2z_1 - z_2 + 3 - i} = \frac{2(1-2i) - (3+2i) + 5 - i}{2(3+2i) - (1-2i) + 3 - i}$$

$$2(1-2i) - (3+2i) + 5 - i = 4 - 7i$$

$$2(3+2i) - (1-2i) + 3 - i = 8 + 5i$$

$$\therefore \frac{2z_2 - z_1 + 5 - i}{2z_1 - z_2 + 3 - i} = \frac{4-7i}{8+5i} = \frac{4-7i}{8+5i} \times \frac{8-5i}{8-5i}$$

$$= \frac{-3-76i}{89} = -\frac{3}{89} - i\,\frac{76}{89}$$

Example 2: If $z_1 = 2+i$, and $z_2 = 3-2i$. Evaluate $\left|\dfrac{2z_2 + z_1 - 5 - i}{2z_1 - z_2 + 3 - i}\right|$ and

express the result in the form of $x + iy$.

Solution

$$\left|\frac{2z_2 + z_1 - 5 - i}{2z_1 - z_2 + 3 - i}\right| = \frac{|2z_2 + z_1 - 5 - i|}{|2z_1 - z_2 + 3 - i|}$$

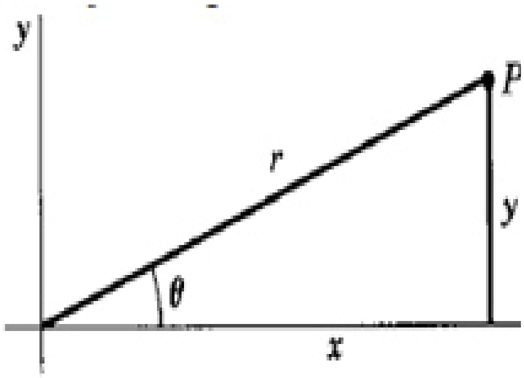
$$2z_2 + z_1 - 5 - i = 3 - 4i$$

$$2z_1 - z_2 + 3 - i = 4 + 3i$$

$$\left|\frac{2z_2 + z_1 - 5 - i}{2z_1 - z_2 + 3 - i}\right| = \frac{|3-4i|}{|4+3i|} = \frac{\sqrt{9+16}}{\sqrt{16+9}} = \frac{\sqrt{25}}{\sqrt{25}} = \frac{5}{5} = 1$$

2- The polar form of complex number

If P is a point in the complex plane corresponding to the complex number $z = x + iy = (x, y)$ Then we see from the figure that .



$$x = r\cos\theta \quad , \quad y = r\sin\theta$$

$$\tan\theta = \frac{y}{x} \Rightarrow\Rightarrow \theta = \tan^{-1}(\frac{y}{x})$$

$$r = \sqrt{x^2 + y^2} = |x + iy| = |z|$$

$$\therefore z = x + iy = r(\cos\theta + i\sin\theta) = re^{i\theta} \qquad (2-1)$$

Equation (2-1) is called the <u>polar form of complex number</u> , where θ : is the angle which $\overline{OP}$ makes with the positive direction of the real axis and it's called the amplitude of complex number z

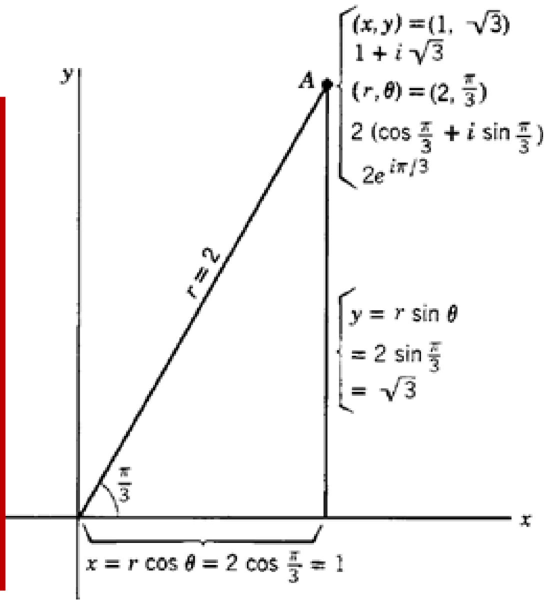
Example (1): Express each of the following complex number in the polar form

(a). $z = 1 + i\sqrt{3}$, **(b).** $z = -1 - i$

Solutions

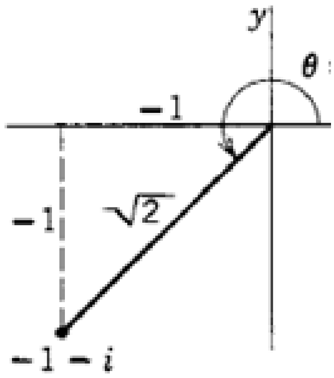
(a)

$$z = 1 + i\sqrt{3} = x + iy = r(\cos\theta + i\sin\theta)$$
$$x = r\cos\theta = 1, \quad y = r\sin\theta = \sqrt{3}$$
$$r = \sqrt{x^2 + y^2} = \sqrt{1+3} = \sqrt{4} = 2$$
$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$$
$$\therefore z = r(\cos\theta + i\sin\theta) = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 2e^{i\frac{\pi}{3}}$$



(b).

$$z = -1 - i = x + iy = r(\cos\theta + i\sin\theta)$$
$$x = r\cos\theta = -1, \quad y = r\sin\theta = -1$$
$$r = \sqrt{x^2 + y^2} = \sqrt{1+1} = \sqrt{2}$$
$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1} 1 = \frac{\pi}{4}$$



but θ **is the angle with the positive direction of the real axis**, so that

$$\theta = \frac{5\pi}{4}$$
$$\therefore z = \sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right) = \sqrt{2}e^{-i\frac{5\pi}{4}}$$