مفردات مادة الفيزياء الرياضية ا – الفصل الأول – العام الدراسي ٢٠٢٠-٢٠٢١

week	Date	Topics Covered
1		Chapter One Complex number, Operation on Complex number Conjugations, Absolute Value
2		Polar Form of Complex number, Demiover's theorem
3		Roots of Complex number, Euler's Formula.
4		Exercises
5		Chapter Two Functions of a complex Variables, Limits, Continuity, Derivatives.
6		Analytic functions, Cauchy- Rieman equation.
7		Elementary Functions, Exponential function, Trigonometric function.
8		Hyperbolic function, Inverse trigonometric function, Inverse Hyperbolic function.
9		Logarithmic function, Power function of complex, Exercises
10		Chapter three Integrations, Contours, Definite integrations.
11		Cauchy's Theorem, Cauchy's integral formulas.
12		Single Point, Residues, The Residue Theorem, Calculation integrals by using The Residue Theorem
13		Exercises
14		Chapter four Series, Converge and diverge, Taylor's Series, Laurent Series
15		Exercises

1. Complex number

To solve an equation of second order

$$ax^2 + bx + c = 0 \quad \text{.....................................(1)}$$

we use the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{..................................(2)}$$

(1) If $b^2 - 4ac \geq 0$, the equation (1) have two real roots (numbers).

(2) If $b^2 - 4ac < 0$, there is no real number x which satisfies the equation (1).

To allows solution of such and similar equation if $b^2 - 4ac < 0$, the set of complex numbers are introduced.

The complex number is written by the form $z = x + iy$, where

x : is the real part of complex number and denoted by [Re(z)].

y : is the real imaginary part of complex number and denoted by [Im(z)].

i : y : is the real imaginary unit , has the property that $i^2 = -1$ and $i = \sqrt{-1}$.

Example : $x^2 + 1 = 0$

Solution: $x = \pm\sqrt{-1} = \pm i$

1.1. Operation on complex numbers

Let $z_1 = x_1 + iy_1$ **, and** $z_2 = x_2 + iy_2$

1- Addition (denoted $z_1 + z_2$**)**

$$z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$$

2- Subtraction (denoted $z_1 - z_2$**)**

$$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$$

3- Multiplication (denoted $z_1 \times z_2$)

$$z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2) = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1)$$

4- Division (denoted $\frac{z_1}{z_2}$, $z_2 \neq 0$)

$$\frac{z_1}{z_2} = \frac{(x_1 + iy_1)}{(x_2 + iy_2)} = \frac{(x_1 + iy_1)}{(x_2 + iy_2)} \times \frac{(x_2 - iy_2)}{(x_2 - iy_2)}$$

$$= \text{.........} = \left(\frac{x_1 x_2 + y_1 y_2}{x^2 + y^2}\right) + i\left(\frac{x_2 y_1 - x_1 y_2}{x^2 + y^2}\right)$$

1.2. Conjugation

If $z = x + iy$, is complex number then its conjugate denoted by $\bar{z} = x - iy$.

Example : if $z = 3 + 2i$, then $\bar{z} = 3 - 2i$

Example: Prove the following properties

(1) $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$, (2) $\overline{z_1 - z_2} = \overline{z_1} - \overline{z_2}$

(3) $\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$, (4) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$

(5) $\mathrm{Re}(z) = \frac{1}{2}(z + \bar{z})$, $\mathrm{Im}(z) = \frac{1}{2i}(z - \bar{z})$

1.3. Absolute value

The absolute value of complex number $z = x + iy$ **denoted by**

$$|z| = \sqrt{z\bar{z}} = \sqrt{x^2 + y^2}$$

Example : If $z = 3 - 4i$

$$|z| = \sqrt{z\bar{z}} = \sqrt{x^2 + y^2} = \sqrt{(3)^2 + (4)^2} = \sqrt{9 + 16} = 5$$

Properties of absolute value

(1) $|z| = \sqrt{z\bar{z}} = \sqrt{x^2 + y^2}$ **, (2)** $|z|^2 = z\bar{z} = x^2 + y^2$ **(3)** $|z_1 . z_2| = |z_1||z_2|$

(4) $\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}, \quad |z_2| \neq 0,$ **(5)** $|z_1 + z_2| \leq |z_1| + |z_2|$