Chapter one

The Slop and The equation of a Straight line

1.The distance between two point

The distance between two points $P_1(x_1, y_1)$ **and** $P_2(x_2, y_2)$ **is given by the following:**

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} \qquad (1)$$

$$\Delta x = x_2 - x_1, \ \Delta y = y_2 - y_1$$

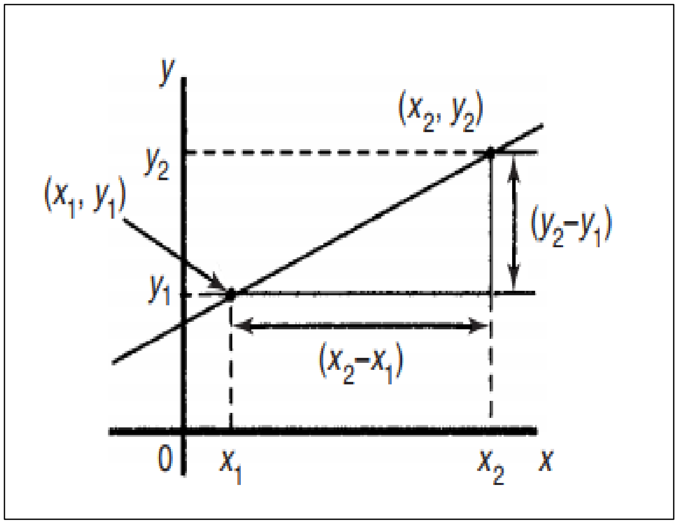


Figure 1

Example: Calculate the distance between the point (-1, 2) and (2, -2).

Solution:

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(2-(-1))^2 + (-2-2)^2} = \sqrt{(3)^2 + (4)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

2- The Slop of a Straight Line

If a straight line is not parallel to the y-axis (x=0), and the straight line graph passing through co-ordinates $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ then the **slope** of a straight line is the ratio of the change in the value of y to the change in the value of x between any two points on the line and is given by:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{...................... ....} \quad (2)$$

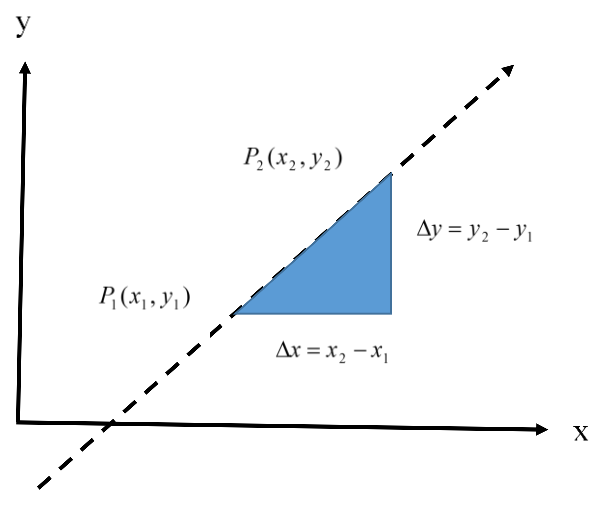


Figure 2

<u>Example:</u> Calculate the slope of the line through the points $P_1(1,2)$ and $P_2(3,8)$.

Solution:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-2}{3-1} = \frac{6}{2} = 3$$

If, as x increases, (→), y also increases (→), then the gradient (slope) is positive. If as x increases (→), y decreases (↓), then the slope is negative. See figure (3):

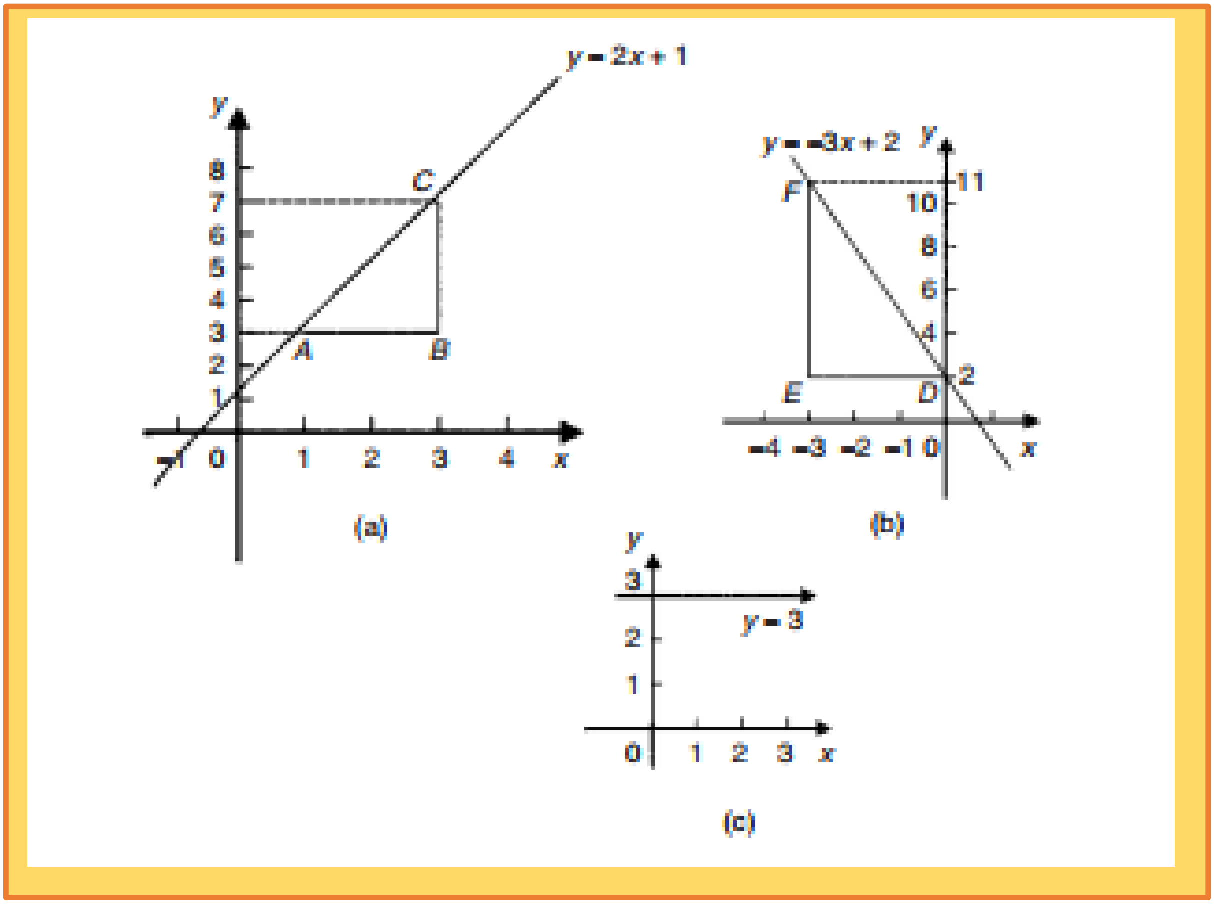


Figure (3)

In figure 3(a), the slope is

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-3}{3-1} = \frac{4}{2} = 2$$

In figure 3(b), the slope is

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11-2}{-3-0} = \frac{9}{-3} = -3$$

Figure 3(c) shows a straight line graph y = 3. Since the straight line is horizontal the gradient (slope) is zero.

The value of y when x = 0 is called the *y*-axis intercept. In Fig. .3(a) the y-axis intercept is 1 and in Fig. .3(b) is 2.

3- Equation of Straight lines

From equation (2) [$m = \frac{\Delta y}{\Delta x} = \frac{y - y_1}{x - x_1}$ **], multiplying both side by (** $x - x_1$**) gives us the more useful equation:**

$$y - y_1 = m(x - x_1) \quad \text{.......................} \quad (3)$$

$$y = mx - mx_1 + y_1$$

$$y = mx + (y_1 - mx_1)$$

$$y = mx + b \quad \text{.......................} \quad (4)$$

Where m is the slope of the line and b is $(y_1 + mx_1)$ **, which is a constant in fact (0,b) is the point where the line crosses the y-axis. The number b is called the y-intercept of the line, and Eq. (4) is called the *slope- intercept equation of the line.***

Example: Write an equation for the line through the point (1,2) with slope $m = -\frac{3}{4}$**. Where does this line cross the y-axis? The x-axis?**

Solution:

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{3}{4}(x - 1)$$

$$y = -\frac{3}{4}x + \frac{3}{4} + 2 = -\frac{3}{4}x + \frac{11}{4}$$

To find where the **line cross the y-axis, we set x=0 in equation above and solve for y:**

$$y = -\frac{3}{4}(0) + \frac{11}{4} = \frac{11}{4}$$

To find where the **line cross the x-axis, we set y=0 in equation above and solve for x:**

$$0 = -\frac{3}{4}x + \frac{11}{4} \Rightarrow \frac{3}{4}x = \frac{11}{4} \Rightarrow x = \frac{11}{3}$$

Example: $y = 5x + 2$**, represents a straight line of gradient (slope) 5 and y-axis intercept 2. Similarly,** $y = -3x - 4$ **represents a straight line of gradient -3 and y-axis intercept -4.**

For a horizontal line the equation $y = mx + b$ **reduces to** $y = 0.x + b$

or $y = b$**, the equation** $y = -5$ **is the slope- intercept equation of the line that passes the through point (0,-5) with slope m=0.**

Functions

Given two sets A and B**,** a set with elements that are ordered pairs ***(***x**,** y***)*,** where x is an element of A and y is an element of B**,** is a relation from A to B**.** A relation from A to B defines a relationship between those two sets. A function is a special type of relation in which each element of the first set is related to exactly one element of the second set. The element of the first set is called the **input**; the element of the second set is called the **output**. Functions are used all the time in mathematics to describe relationships between two sets. For any function, when we know the input, the output is determined, so we say that the output is a function of the input. For example, the area of a square is determined by its side length, so we say that the area (the output) is a function of its side length (the input). The velocity of a ball thrown in the air can be described as a function of the amount of time the ball is in the air. Since functions have so many uses, it is important to have precise definitions and terminology to study them.

Definition

A function ***f*** consists of a set of inputs, a set of outputs, and a rule for assigning each input to exactly one output. The set of inputs is called the domain of the function. The set of outputs is called the range of the function.

The concept of a function can be visualized using **Figure 2.1**, **Figure 2.2**, and **Figure 2.3**.

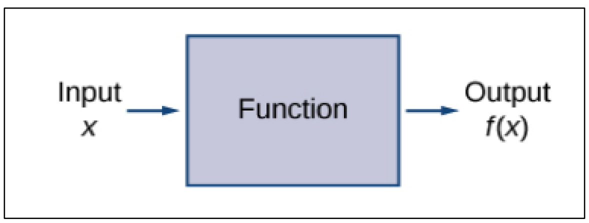


Figure 2.1 A function can be visualized as an input/output device.

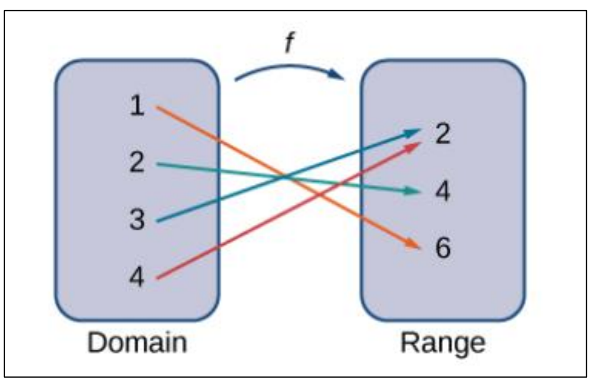


Figure 2.2 A function maps every element in the domain to exactly one element in the range.

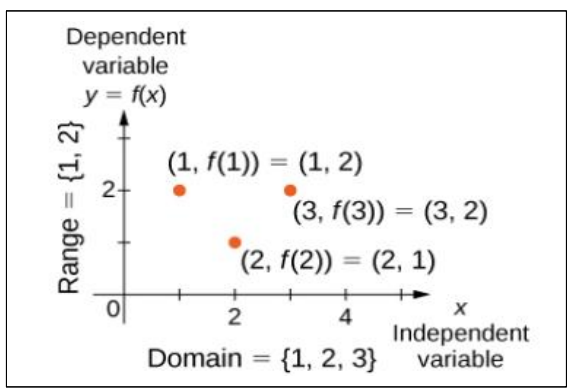


Figure 2.3 In this case, a graph of a function f has a domain of ***{1, 2, 3}*** and a range of ***{1, 2}*.** The independent variable is x and the dependent variable is y**.**

Every function is determined by two things: (1) the domain of the first variable x and (2) the Range or the rule or condition that the pairs (x, y) must satisfy to belong the function.

To graph a function, we carry out three steps.

1.Make a table of pairs from the function.

2.Plot enough of corresponding point to learn the shape of graph. Add more pairs to the table if necessary.

3.Complete the sketch by connecting the points.

Example:

Problem 1. Plot the graph $f(x) = y = 4x + 3$ in the range x = -3 to x = +4. From the graph, find (a) the value of y when x = 2.2, and (b) the value of x when y = -3

Solution:

The domain is taken to be the set of all real numbers *x* for which *f* (*x*) is a real number

D $f(x)$ =R

To find the range we set

$$f(x) = y = 4x + 3 \Rightarrow 4x = y - 3$$
$$\Rightarrow x = \frac{y-3}{4}$$

So that the range is taken to be the set of all real numbers y.

R $f(x)$ =R

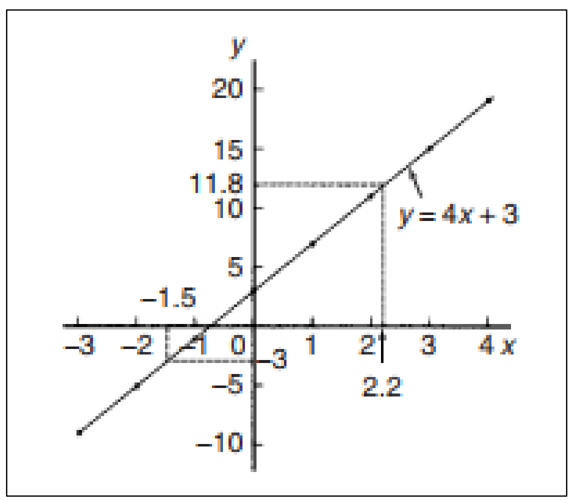
Whenever an equation is given and a graph is required, a table giving corresponding values of the variable is necessary. The table is achieved as follows:

When $x = -3,\ y = 4x + 3 = 4(-3) + 3$
$= -12 + 3 = -9$

When $x = -2,\ y = 4(-2) + 3$
$= -8 + 3 = -5$, and so on.

Such a table is shown below:

x	-3	-2	-1	0	1	2	3	4
y	-9	-5	-1	3	7	11	15	19



The co-ordinates (3, 9), (2, 5), (1, 1), and so on, are plotted and joined together to produce the straight line shown in Figure above (Note that the scales used on the x and y axes do not have to be the same). From the graph:

(a) when x = 2.2, $y = 11.8$**, and**
(b) when y D = -3, $x = -1.5$

Example: Consider the function

$$f(x) = y = \sqrt{x+3} + 1$$

find the following: 1.The domain. 2.The range. 3. Sketch a graph of $f(x)$

Solution:

1.To find the domain we set

$$\sqrt{x+3} \geq 0 \Rightarrow x+3 \geq 0 \Rightarrow x \geq -3$$

D $f(x) = \{x : x \geq -3\}$

2. To find the range we set

$$y = \sqrt{x+3} + 1$$
$$y - 1 = \sqrt{x+3}$$
$$(y-1)^2 = x+3$$
$$x = (y-1)^2 - 3$$

We just need to verify that x is in the domain of $f(x)$. Since the domain of f consists of all real numbers greater than or equal to -3, and

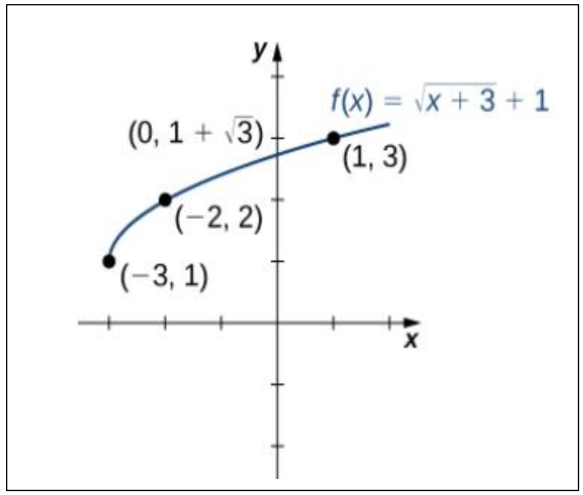
$$x = (y-1)^2 - 3 \geq -3$$

there does exist an x in the domain of $f(x)$. We conclude that the range of f is $\{y : y \geq -1\}$

R $f(x) = \{y : y \geq -1\}$

3. To graph this function, we make a table of values. Since we need $x + 3 \geq 0$, we need to choose values of $x \geq -3$. We choose values that make the square-root function easy to evaluate.

x	-3	-2	1
$f(x)$	1	2	3



Example: **Consider the function**

$$f(x) = y = \sqrt{1 - x^2}$$

find the following: 1. The domain. 2.The range. 3. Sketch a graph of $f(x)$

Solution:

1.To find the domain we set

$$\sqrt{1 - x^2} \ge 0 \Rightarrow 1 - x^2 \ge 0$$

$$(1 - x)(1 + x) \ge 0$$

$$(1 - x) \ge 0 \Rightarrow -x \ge -1 \Rightarrow\Rightarrow x < 1$$

$$or$$

$$(1 + x) \ge 0 \Rightarrow\Rightarrow x \ge -1$$

So that

D $f(x) = \{x : -1 \le x \le 1\}$

2. To find the range we set

$$y = \sqrt{1 - x^2} \Rightarrow y^2 = 1 - x^2 \Rightarrow x = \sqrt{1 - y^2}$$

$$\sqrt{1-y^2} \geq 0$$

$$\sqrt{1-y^2} \geq 0 \Rightarrow 1-y^2 \geq 0$$

$$(1-y)(1+y) \geq 0$$

$$(1-y) \geq 0 \Rightarrow -y \geq -1 \Rightarrow\Rightarrow y < 1$$

$$or$$

$$(1+y) \geq 0 \Rightarrow\Rightarrow y \geq -1$$

So that

R $f(x) =$ $\{y : -1 \leq y \leq 1\}$

3. Exercise: (is left to the student as homework)

Example. Consider the function $f(x) = y = 1/\sqrt{3-x}$ **, find the following:**

1.The domain. **2.The range.** **3.** Sketch a graph of $f(x)$

1.To find the domain we set

$$\sqrt{3-x} > 0 \Rightarrow -x > -3 \Rightarrow\Rightarrow x < 3$$

The set of real number less than 3

D $f(x) =$ $\{x : x < 3\}$

2. To find the range we set

$$y = \frac{1}{\sqrt{3-x}} \Rightarrow \sqrt{3-x} = \frac{1}{y} \Rightarrow 3-x = \frac{1}{y^2} \Rightarrow x = 3 - \frac{1}{y^2}$$

We just need to verify that x is in the domain of $f(x)$. Since the domain of f consists of all real numbers **less than 3**, and

$$x = 3 - \frac{1}{y^2} < 3$$

there does exist an x in the domain of $f(x)$. We conclude that the range of f is $\{y : y > 0\}$