Chapter one

The Slop and The equation of a Straight line

1.The distance between two point

The distance between two points $P_1(x_1, y_1)$ **and** $P_2(x_2, y_2)$ **is given by the following:**

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} \qquad (1)$$

$$\Delta x = x_2 - x_1, \ \Delta y = y_2 - y_1$$

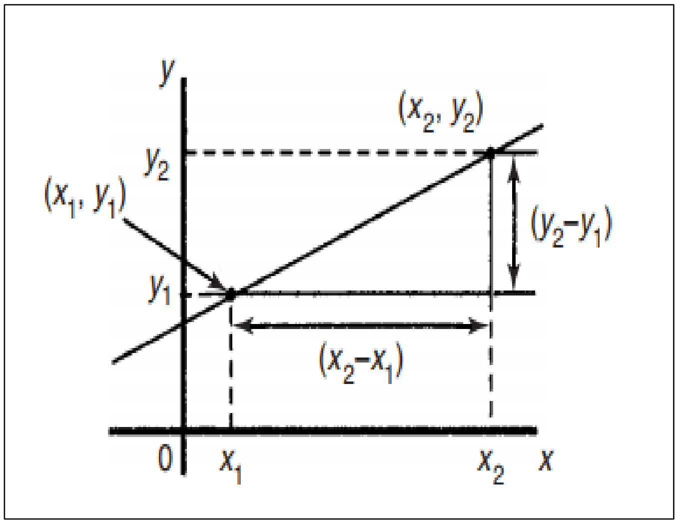

Figure 1

Example: Calculate the distance between the point (-1, 2) and (2, -2).

Solution:

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(2-(-1))^2 + (-2-2)^2} = \sqrt{(3)^2 + (4)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

2- The Slop of a Straight Line

If a straight line is not parallel to the y-axis (x=0), and the straight line graph passing through co-ordinates $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ then the **slope** of a straight line is the ratio of the change in the value of y to the change in the value of x between any two points on the line and is given by:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

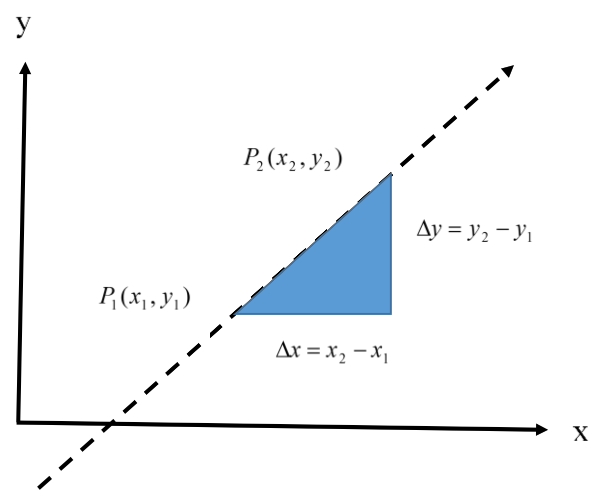

Figure 2

<u>Example:</u> Calculate the slope of the line through the points P₁(1,2) and P₂(3,8).

Solution:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-2}{3-1} = \frac{6}{2} = 3$$

If, as x increases, (→), y also increases (→), then the gradient (slope) is positive. If as x increases (→), y decreases (↓), then the slope is negative. See figure (3):

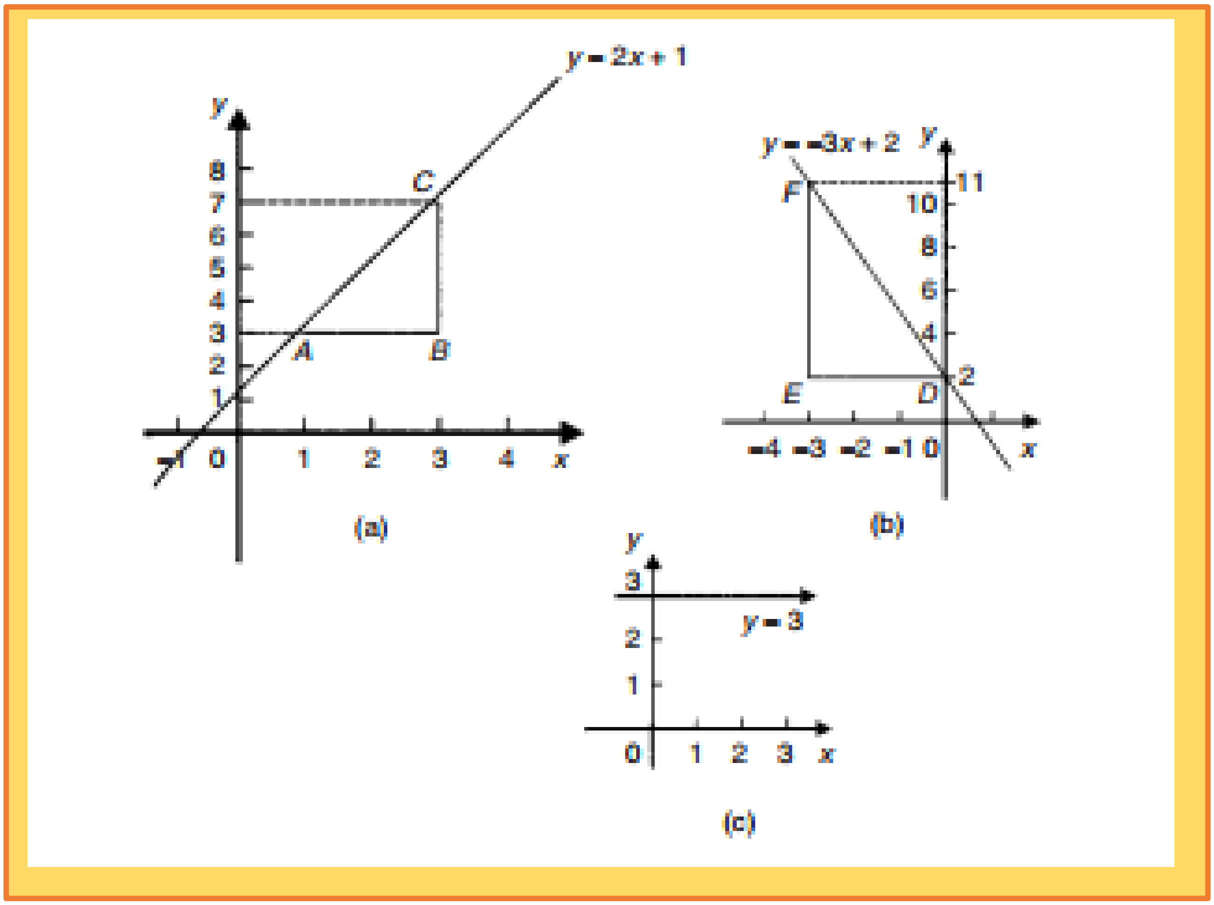

Figure (3)

In figure 3(a), the slope is

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-3}{3-1} = \frac{4}{2} = 2$$

In figure 3(b), the slope is

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11-2}{-3-0} = \frac{9}{-3} = -3$$

Figure 3(c) shows a straight line graph y = 3. Since the straight line is horizontal the gradient (slope) is zero.

The value of y when x = 0 is called the *y*-axis intercept. In Fig. .3(a) the y-axis intercept is 1 and in Fig. .3(b) is 2.

3- Equation of Straight lines

From equation (2) [$m = \dfrac{\Delta y}{\Delta x} = \dfrac{y - y_1}{x - x_1}$ **], multiplying both side by (** $x - x_1$**) gives us the more useful equation:**

$$y - y_1 = m(x - x_1) \quad \text{.......... (3)}$$

$$y = mx - mx_1 + y_1$$

$$y = mx + (y_1 - mx_1)$$

$$y = mx + b \quad \text{.......... (4)}$$

Where m is the slope of the line and b is $(y_1 + mx_1)$ **, which is a constant in fact (0,b) is the point where the line crosses the y-axis. The number b is called the y-intercept of the line, and Eq. (4) is called the *slope- intercept equation of the line.***

Example: Write an equation for the line through the point (1,2) with slope $m = -\frac{3}{4}$**. Where does this line cross the y-axis? The x-axis?**

Solution:

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{3}{4}(x - 1)$$

$$y = -\frac{3}{4}x + \frac{3}{4} + 2 = -\frac{3}{4}x + \frac{11}{4}$$

To find where the **line cross the y-axis, we set x=0 in equation above and solve for y:**

$$y = -\frac{3}{4}(0) + \frac{11}{4} = \frac{11}{4}$$

To find where the **line cross the x-axis, we set y=0 in equation above and solve for x:**

$$0 = -\frac{3}{4}x + \frac{11}{4} \Rightarrow \frac{3}{4}x = \frac{11}{4} \Rightarrow x = \frac{11}{3}$$

Example: $y = 5x + 2$**, represents a straight line of gradient (slope) 5 and y-axis intercept 2. Similarly,** $y = -3x - 4$ **represents a straight line of gradient -3 and y-axis intercept -4.**

For a horizontal line the equation $y = mx + b$ **reduces to** $y = 0.x + b$

or $y = b$**, the equation** $y = -5$ **is the slope- intercept equation of the line that passes the through point (0,-5) with slope m=0.**