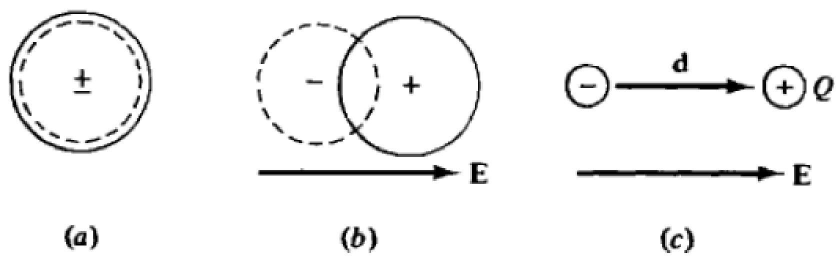
Dielectric Material

Polarization:

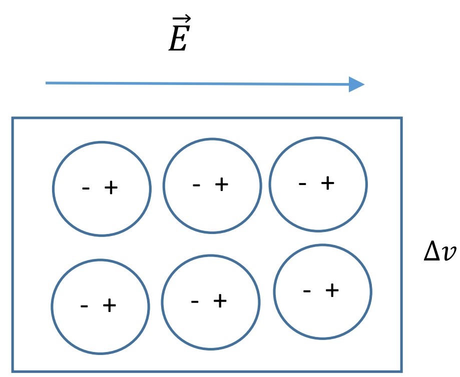
Dielectric materials become polarized in an electric field with the result that the electric flux density **D** is greater than it would be under free space conditions with the same field intensity.



As result from the figure we can get:

$$p = Q\ d$$

p the electric dipole moment.



Polarization is defined as dipole moment per unit volume.

$$\vec{P} = \lim_{\Delta v \to 0} \frac{Np}{\Delta v} \qquad C/m^2$$

$$D = \varepsilon_0 E + P$$

This equation permits **E** and **P** to have different directions.

In an isotropic or linear material **E** and **P** are parallel at each point which is expressed by:

$$P = x_e \varepsilon_0 E$$

x_e electric susceptibility

$$D = \varepsilon_0 (1 + x_e) E = \varepsilon_0 \varepsilon_r E$$

$\varepsilon_r = (1 + x_e)$ is also a pure number. Since $D = \varepsilon E$ $\quad \varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$

ε_r is called relative permittivity.

1) Find the magnitude of D and P for a dielectric field material in which E=0.15 MV/m and x_e=4.25 .

$$\varepsilon_r = 1 + x_e = 1 + 4.25 = 5.25$$

$$D = \varepsilon_0 \varepsilon_r E = \frac{10^{-9}}{36\pi} \cdot 5.25 \cdot (0.15 \times 10^6) = 6.96\ \frac{\mu\, C}{m^2}$$

$$P = x_e \varepsilon_0 E = 4.25 \cdot \frac{10^{-9}}{36\pi} \cdot (0.15 \times 10^6) = 5.64\ \frac{\mu\, C}{m^2}$$

2) Find polarization in dielectric material with $\varepsilon_r = 2.8$ if $D = 3 \times 10^{-7} C/m^2$.

$$P = x_e \varepsilon_0 E$$

$$\varepsilon_r = 1 + x_e \qquad x_e = 2.8 - 1 = 1.8$$

$$D = \varepsilon_0 \varepsilon_r E \qquad E = \frac{D}{\varepsilon_0 \varepsilon_r}$$

$$P = 1.8 \cdot \varepsilon_0 \cdot \frac{3 \times 10^{-7}\ C/m^2}{\varepsilon_0 (2.8)} = 1.92 \times 10^{-7}\ C/m^2$$

Boundary conditions at interface of two dielectric materials.

1)The tangential component is continues across a dielectric interface.

$$E_{t1} = E_{t2} \qquad and\ \frac{D_{t1}}{\varepsilon r_1} = \frac{D_{t2}}{\varepsilon r_2}$$

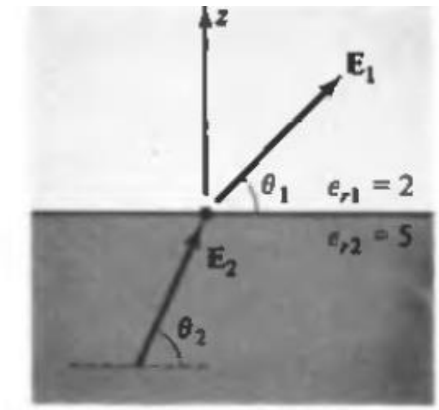
2)The normal component of D has discontinuity of magnitude $|\rho_s|$ across a dielectric interface . If the unit normal vector is chosen to point into dielectric 2, then this condition can be written:

$$D_{n1} - D_{n2} = -\rho_s \qquad and \qquad \varepsilon_{r1} E_{n1} - \varepsilon_{r2} E_{n2} = -\frac{\rho_s}{\varepsilon_0}$$

Generally the interface will have no free charge $\rho_s = 0$ so that,

$$D_{n1} = D_{n2} \qquad \varepsilon_{r1} E_{n1} = \varepsilon_{r2} E_{n2}$$

1) Given that $E_1 = 2a_x - 3a_y + 5a_z \;\; V/m$ at the charge free dielectric interface of figure .Find **D**2 and the angles $\theta1 \; and \; \theta2$.



$$E_1 = 2a_x - 3a_y + 5a_z$$

$$E_2 = 2a_x - 3a_y + E_{z2}a_z$$

$$D_1 = \varepsilon_0\varepsilon_{r1}E_t = 4\varepsilon_0 a_x - 6\varepsilon_0 a_y + 10\varepsilon_0 a_z$$

$$D_2 = D_{x2}a_x - D_{y2}a_y + 10\varepsilon_0 a_z$$

$$D_2 = \varepsilon_0\varepsilon_{r2}E_2$$

$$D_{x2}a_x - D_{y2}a_y + 10\varepsilon_0 a_z = 2\varepsilon_0\varepsilon_{r2}a_x - 3\varepsilon_0\varepsilon_{r2}a_y + \varepsilon_0\varepsilon_{r2}E_{z2}a_z$$

$$D_{x2} = 10\varepsilon_0 \quad D_{y2} = -15\varepsilon_0 \qquad E_{z2} = \frac{10}{\varepsilon_{r2}} = 2$$

$$E_1 \cdot a_z = |E_1| \cos(90 - \theta1)$$

$$5 = \sqrt{38} \sin\theta1$$

$$E_2 \cdot a_z = |E_2| \cos(90 - \theta2)$$

$$2 = \sqrt{17} \sin\theta2$$

$$\theta1 = 54.2° \qquad \theta2 = 29.0°$$

A useful relation can be obtained from

$$\tan\theta1 = \frac{E_{z1}}{\sqrt{E_{x1}^2 + E_{y1}^2}} = \frac{D_{z1}/\varepsilon_0\varepsilon_{r1}}{\sqrt{E_{x1}^2 + E_{y1}^2}}$$

$$\tan\theta2 = \frac{E_{z2}}{\sqrt{E_{x2}^2 + E_{y2}^2}} = \frac{D_{z2}/\varepsilon_0\varepsilon_{r2}}{\sqrt{E_{x2}^2 + E_{y2}^2}}$$

$$\frac{\tan\theta1}{\tan\theta2} = \frac{\varepsilon_{r2}}{\varepsilon_{r1}}$$

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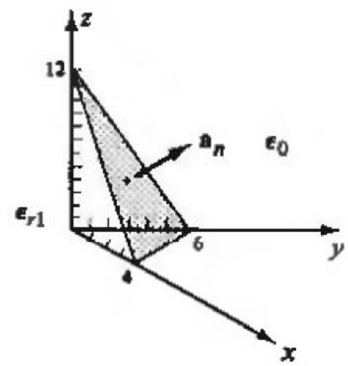
2) In the free space region x<0 the electric field intensity is $E_1 = 3a_x + 5a_y - 3a_z \;\; V/m$. The region x>0 is a dielectric for which $\varepsilon_{r2} = 3.6$. Find the angle $\theta2$ that the field in the dielectric makes with x=0 plane.

$$E_1 \cdot a_x = |E_1| \cos(90 - \theta1)$$

$3 = \sqrt{43}\sin\theta 1 \qquad \theta 1 = 27.2°$

$\tan\theta 2 = \frac{1}{\varepsilon_{r2}}\tan\theta 1 = 0.1428 \qquad \theta 2 = 8.13°$

3) A dielectric free space interface has the equation $3x + 2y + z = 12\ m$ the origin of the interface has $\varepsilon_{r1} = 3\ \ and\ \ E_1 = 2a_x + 5a_z\ \frac{V}{m}$. Find E₂



$$a_n = \frac{3a_x + 2a_y + a_z}{\sqrt{14}}$$

The projection of E_1 on a_n is the normal component of E at the interface.

$$E_1 \cdot a_n = \frac{11}{\sqrt{14}}$$

$$E_{n1} = \frac{11}{\sqrt{14}}\left(\frac{3a_x + 2a_y + a_z}{\sqrt{14}}\right) = 2.36a_x + 1.57a_y + 0.79a_z$$

$$E_{t1} = E_1 - E_{n1} = -0.36a_x - 1.57a_y + 4.21a_z = E_{t2}$$

$$D_{n1} = \varepsilon_0\varepsilon_{r1}E_{n1} = \varepsilon_0(7.08a_x + 4.71a_y + 2.37a_z) = D_{n2}$$

$$E_{n2} = \frac{1}{\varepsilon_0}D_{n2} = 7.08a_x + 4.71a_y + 2.37a_z$$

$$E_2 = E_{t2} + E_{n2} = 6.72_x + 3.14a_y + 6.58a_z$$

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