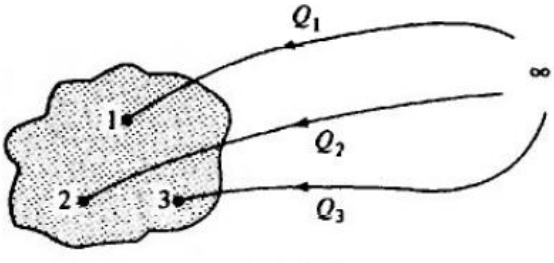
Energy stored in static electric field :

We suppose three charge in free region with E=0, the total work of these charges is :



$W_E = W_1 + W_2 + W_3$

$W_E = 0 + Q_2 V_{2,1} + (Q_3 V_{3,1} + Q_3 V_{3,2})$

Where W_E is the energy stored in the electric field of the charge distribibition.

Now if the three charge were brought into place in revers order,the total work would be

$W_E = W_3 + W_2 + W_1$

$W_E = 0 + Q_2 V_{2,3} + (Q_1 V_{1,2} + Q_1 V_{1,3})$

When the tow result above added, we obtained the twice of stored energy.

$2W_E = Q_1(V_{1,2} + V_{1,3}) + Q_2(V_{2,1} + V_{2,3}) + Q_3(V_{3,1} + V_{3,2})$

The term $Q_1(V_{1,3} + V_{1,2})$ was the work done aganist the electric fields of Q_2 and Q_3 so $(V_{1,2} + V_{1,3}) = V_1$ the potential at position 1 then :

$2W_E = Q_1 V_1 + Q_2 V_2 + Q_3 V_3$

$W_E = \frac{1}{2}\sum_{m=1}^{n} Q_m V_m$ For a region containing n point charge

$W_E = \frac{1}{2}\int \rho V \, dv$ For a region with a charge density ρ $(\frac{C}{m^3})$

$W_E = \frac{1}{2}\int D.E \, dv$ $W_E = \frac{1}{2}\int \varepsilon E^2 \, dv$ $W_E = \frac{1}{2}\int \frac{D^2}{\varepsilon} \, dv$ Other forms

In an electric circuit the energy stored in the field of a capacitor is given by :

$$W_E = \frac{1}{2} QV = \frac{1}{2} CV^2$$

1. What energy stored in system of two point charges $Q_1 = 3\ nC; Q_1 = -3\ nC$ separated by d=0.2 m ?

$$2W_E = Q_1V_1 + Q_2V_2$$

$$2W_E = Q_1\left(\frac{Q_2}{4\pi\varepsilon_0 d}\right) + Q_2\left(\frac{Q_1}{4\pi\varepsilon_0 d}\right)$$

$$2W_E = \frac{2Q_1Q_2}{4\pi\varepsilon_0 d} = \frac{(-3\times10^{-9})\times(3\times10^{-9})}{4\pi(\frac{10^{-9}}{36\pi})(0.2)} = -405\ nJ$$

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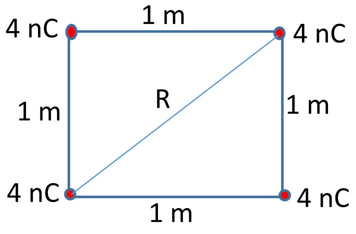
2. What energy stored of four point charges 4 nC seperated by d=1 m ?



$$2W_E = Q_1V_1 + Q_2V_2 + Q_3V_3 + Q_4V_4$$

$$2W_E = 4Q_1V_1$$

$$2W_E = 4Q_1\left(\frac{Q}{4\pi\varepsilon_0}\right)\left(\frac{1}{1}+\frac{1}{1}+\frac{1}{\sqrt{2}}\right)$$

$$W_E = \left(\frac{8\times10^{-9}\times4\times10^{-9}}{4\pi(\frac{10^{-9}}{36\pi})}\right)\left(\frac{1}{1}+\frac{1}{1}+\frac{1}{\sqrt{2}}\right)$$

$$W_E = 780\ nJ$$

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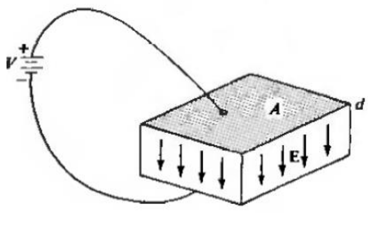
3. A parallel plate capacitor , for which $C = \frac{\varepsilon A}{d}$ has a constant voltageV applied acrosse the plates. Find the stored energy in the electric field .



$E = (V/d)_{a_n}$ between the plates and E=0 elsewhere.

$$W_E = \frac{1}{2}\int \varepsilon E^2\, dv$$

$$W_E = \frac{\varepsilon}{2}\left(\frac{V}{d}\right)^2 \int dv$$

$$W_E = \frac{\varepsilon}{2}\frac{AV^2}{d}$$

$$W_E = \frac{1}{2}CV^2$$

****

Current, Current density and Conductors

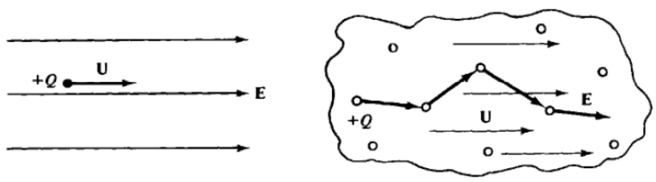
Electric current is the rate of transport of electric charge past a specified point or across specific surface..

$$I = \frac{Q}{t}\ \frac{C}{sec} = Ampere$$

Charges in Motion

Consider the force on positively charged particle in an electric field in vacuum, as shown in figure. This force $F = +QE$ is unopposed and results in constant acceleration. Thus the charge moves in the direction of E with a velosity U that increases as long as the particle in the E field.

$U = \mu E$ The most elementary expression of Ohm's law



(a) Vacuum (b) Liquid or gas

μ mobility has unit $m^2/V.s$ and U drift velosity.

In terms of the basic properties of the right-cylindrical current-carrying metallic ohmic conductor, where the charge-carriers are electrons, this expression can be rewritten as:

$$U = \frac{m\sigma\Delta V}{\rho e f l}$$

- U is again the drift velocity of the electrons, in $m\cdot s^{-1}$
- m is the molecular mass of the metal, in kg

- σ is the electric conductivity of the medium at the temperature considered, in S/m.
- ΔV is the voltage applied across the conductor, in V
- ρ is the density (mass per unit volume) of the conductor, in kg·m^{-3}
- *e is the elementary charge, in C*

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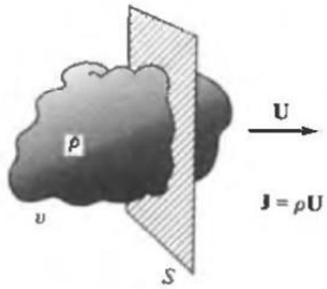
Convection current density J

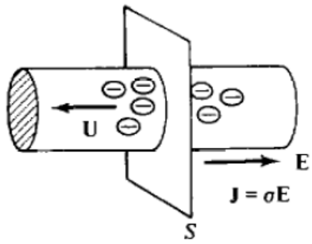
If we have particle with density charge ρ in a volume v as in figure. As this charge configuration passes a surface S it constitutes a convection current with density.

$$J = \rho U \ \ (\frac{A}{m^2})$$



Conduction current density J

The conduction current density that occurs in the presence of an electric field within a conductor of fixed cross section.



From $J = \rho U \quad and \quad U = \mu E$

$J = \rho \mu E$

$J = \sigma E$

$\sigma = \rho \mu \ \ S/m$ is the conductivity of the material.

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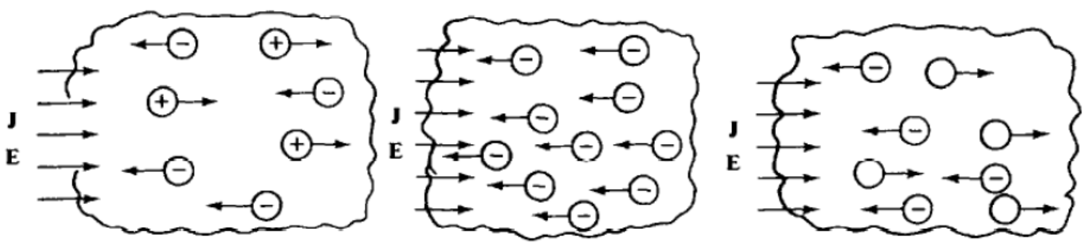
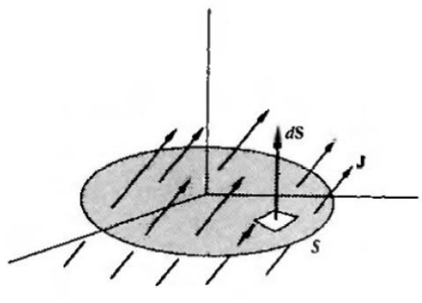
Conductivity $\boldsymbol{\sigma}$:



Gas or liquid	**Conductor**	**Semiconductor**
$\boldsymbol{\sigma = \rho^- \mu^- + \rho^+ \mu^+}$	$\boldsymbol{\sigma = \rho_e \mu_e}$	$\boldsymbol{\sigma = \rho_e \mu_e + \rho_h \mu_h}$

Current I :

If the current density J crosses a surface S, as in figure , the current I obtained by the integration the dot product of J and ds.



$$dI = J.ds \qquad I = \int_s J.ds$$

****

1. What electric field intensity and current density correspond to drift velosity of $6 \times 10^{-4}\ m/s$ in a silver conductor ? for silver $\sigma = 61.7 \frac{MS}{m}$ $and\ \mu = 5.6 \times 10^{-3}\ \frac{m^2}{V.s}$

$$E = \frac{U}{\mu} = \frac{6 \times 10^{-4}}{5.6 \times 10^{-3}} = 1.07 \times 10^{-1}\ \ V/m$$

$$J = \sigma E = 61.7 \times 10^6 \times 1.07 \times 10^{-1} = 6.61 \times 10^6\ \ A/m^2$$

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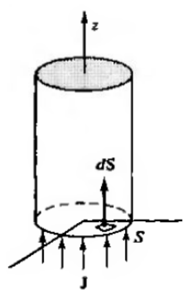
2. Find the current in the circular wire as in figure if the current density is J=15(1-e^{-1000})a_z (A/m^2). The radius of the wire is 2mm.



$$dI = J.ds$$

$$dI = 15(1 - \mathrm{e}^{-1000}) a_z \cdot rdr\ d\emptyset\ a_z$$

$$I = \int_0^{2\pi} \int_0^{0.002} 15(1 - \mathrm{e}^{-1000})\ rdr\ d\emptyset$$

$$I = 1.33 \times 10^{-4}\ A = 0.133\ A$$

****

3. Find the current crossing the portion of the x=0 plane defined by $-\frac{\pi}{4} \leq y \leq \frac{\pi}{4}$ $, -0.01 \leq z \leq 0.01\ m$ if $J = 100\cos 2y\ a_x\ A/m^2$?

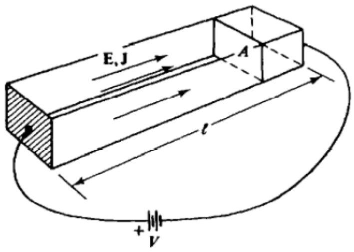
$$I = \int J.ds$$

$$I = \int_{-0.01}^{0.01} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 100\cos 2y\ a_x \cdot dy\ dz\ a_x = 2\ A$$

****

Resistance R

If a conductor of uniform crosse sectional area A and length L as in figure has a voltage difference V between its ends then



$$E = \frac{V}{L} \quad and \quad J = \frac{\sigma V}{L}$$

Assuming that the current is uniformly distributed over the area A. The total current is then

$$I = J\ A = \frac{\sigma\ A\ V}{L}$$

Since Ohm's law states that V=IR, the resistance is

$$R = \frac{L}{\sigma\ A} \quad \Omega$$

For nonuniform current distributions the resistance is given by :

$$R = \frac{V}{\int J \cdot ds} = \frac{V}{\int \sigma E \cdot ds}$$

$$R = \frac{\int E \cdot dl}{\int \sigma E \cdot ds}$$

****

Continuity of current

$$\oint J.ds = I = \frac{-dQ}{dt} = \frac{-\partial}{\partial t}\int \rho\, dv$$

$$\frac{\oint J \cdot ds}{\Delta v} = \frac{-\partial}{\partial t}\int \frac{\rho\, dv}{\Delta v}$$

$$\overrightarrow{\nabla} \cdot \vec{J} = \frac{-\partial \rho}{\partial t}$$

$$\overrightarrow{\nabla} \cdot \vec{J} = 0 \quad \textbf{Kirchhoff Current Law}$$

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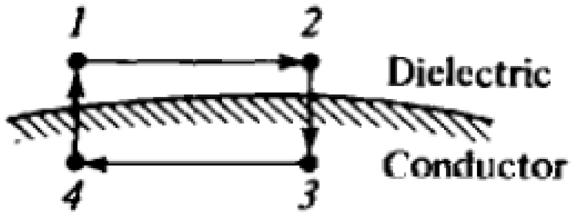
Conductor dielectric boundary conditions.

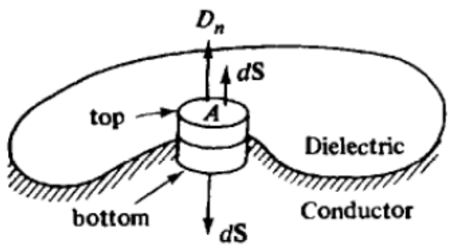
Under static condition all the net charge outer surface of conductor and both E and D are therefore zero within the conductor. Because the electric fiel dis conservative



$$\oint \boldsymbol{E} \cdot \boldsymbol{dl} = \boldsymbol{0}$$

$$\oint_1^2 \boldsymbol{E} \cdot \boldsymbol{dl} + \oint_2^3 \boldsymbol{E} \cdot \boldsymbol{dl} + \oint_3^4 \boldsymbol{E} \cdot \boldsymbol{dl} + \oint_4^1 \boldsymbol{E} \cdot \boldsymbol{dl}$$

$$\oint_1^2 \boldsymbol{E} \cdot \boldsymbol{dl} = \boldsymbol{0} \qquad \boldsymbol{E_t} = \boldsymbol{D_t} = \boldsymbol{0}$$



$$\oint \boldsymbol{D} \cdot \boldsymbol{ds} = \boldsymbol{Q}$$

$$\int_{top} \boldsymbol{D} \cdot \boldsymbol{ds} + \int_{bottom} \boldsymbol{D} \cdot \boldsymbol{ds} + \int_{slide} \boldsymbol{D} \cdot \boldsymbol{ds}$$

$$= \int_{A} \boldsymbol{\rho_s} \cdot \boldsymbol{ds}$$

$$\boldsymbol{D_t} = \boldsymbol{\rho_s} \qquad \boldsymbol{E_t} = \frac{\boldsymbol{\rho_s}}{\boldsymbol{\varepsilon}}$$

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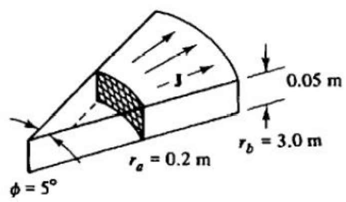
Exemple : Find the resistance between the inner and the outer curved surfaces of the block as in figure where the material is silver for which $\sigma = 6.17 \times 10^{7}\ S/m$?



$$J = \frac{k}{r}\ a_r \quad and \quad E = \frac{k}{\sigma r} a_r$$

$$R = \frac{\int_{0.2}^{0.3} \frac{k}{\sigma r} a_r \cdot dr a_r}{\int_{0}^{0.05} \int_{0}^{0.0873} \frac{k}{r} a_r \ \cdot r\ d\emptyset\ dz\ a_r}$$

$$R = \frac{\ln 15}{\sigma(0.05)(0.0873)} = 1.01\ \times 10^{-5}\Omega = 10.1\ \mu\Omega$$

****