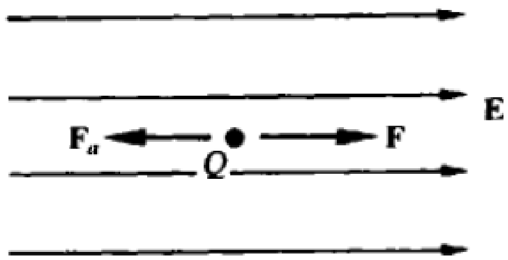
Energy and electric potential of charge system

A charge Q experiences a force F in an electric field E. In order to maintain the charge in equilibrium a force F_a must be applied in opposition.



$F = Q\ E\ ; \qquad F_a = -QE$

Where the work is defined as a force acting over a distance. Therefore, differential amount of work dw is done when the applied force F_a produces as a differential displacement dl of the charge .

$dw = F_a \cdot dl = -QE \cdot dl$

Note:

When Q is positive and dl is in the direction, $dw = -QEdl \; < 0$ this mean the work was done by the electric field. On the other hand, when the work is positive this mean the work done against the electric field.

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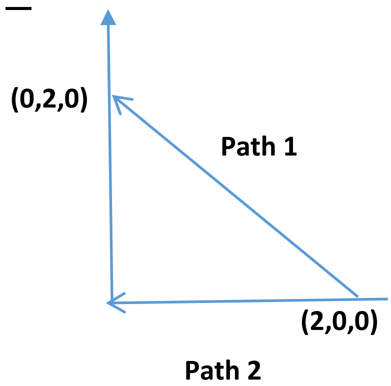
1. Find the work done in moving a charge of +2 C from (2,0,0) to (0,2,0) along straight line as in figure if $\vec{E} = 2x\ a_x - 4ya_y \;\; V/m$?



$dw = -QE.dl$

$dw = -2(2x\ a_x - 4ya_y\ )\ .(dx\ a_x + dy\ a_y\ + dz\ a_z\ )$

$dw = -4x\ dx + 8y\ dy$

$x + y = 2 \quad path\ equation$

$y = 2 - x \qquad dy = -dx$

$dw = -4x\ dx + 8(2 - x)\ (-dx)$

$dw = 4xdx - 16dx$

$$w = \int_{2}^{0} (4x - 16)dx = 24 \ J$$

For verification the result you can found w1 and w2, then w=w1+w2

It should be the same result.

Find $\oint E.dl$???? H.W

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Electric potential between two points

The potential of point A with respect to point B is define as the work done in moving a unit positive charge Q_u from B to A.

$$V_{AB} = \frac{W}{Q} = -\int_{B}^{A} E \cdot dl$$

1. Find the potential of A$(1, \emptyset, z)$ with respect to B $(3, \grave{\emptyset}, \grave{z})$ in cylindrical coordinate where electric field due to line charge on Z-axes is given by $\vec{E} = \frac{50}{r} a_r \ V/m$?

$$V_{AB} = -\int_{B}^{A} E \cdot dl \qquad ; \qquad V_{AB} = -\int_{B}^{A} E_r \cdot dr$$

$$= -\int_{1}^{3} \frac{50}{r} \, dr = -50 \ln r|_1^3 = -50 \ln \frac{1}{3} = 54.9 \, V$$

The work of one-point charge:

$$V_{AB} = \frac{W}{Q} = -\int_{B}^{A} E \cdot dl$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$V_{AB} = -\int \frac{Q}{4\pi\varepsilon_0 r^2}\, dl \qquad ; dl = dr$$

$$V_{AB} = -\frac{Q}{4\pi\varepsilon_0}\int_{r_A}^{r_B} \frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right) \quad ; if\ r_B = \infty$$

$$V_{AB} = \frac{Q}{4\pi\varepsilon_0 r}$$

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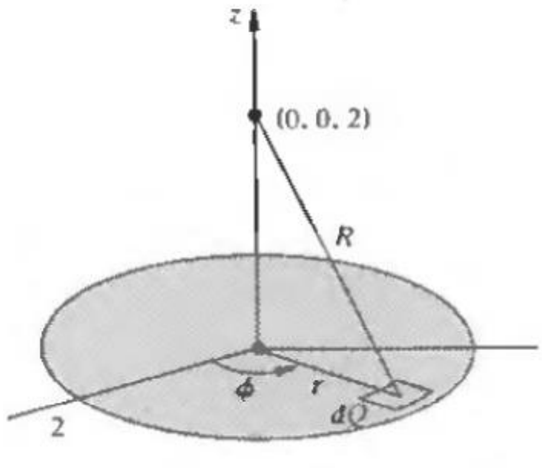
1. Charge $\left(\frac{40}{3}\right) nC$ is uniformly distributed around ring of radius (2 m) Find the potential at point on axis (2m) from the plane of ring?

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

$$V = \int \frac{\rho_s\, ds}{4\pi\varepsilon_0 r}$$

$$\rho_s = \frac{Q}{A} = \frac{\frac{40}{3} \times 10^{-9}}{2\pi r} = \frac{10^{-8}}{3\pi}\ C/m$$



$$dQ = \rho_s\, ds \quad ; ds = r\, dr\, d\emptyset$$

$$R = \sqrt{r^2 + 4}$$

$$V = \frac{30}{\pi}\int_0^{2\pi}\int_0^{2} \frac{r\, dr\, d\emptyset}{\sqrt{r^2 + 4}} = 49.7\ V$$

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2. Given the field $E = \left(-\frac{16}{r^2}\, a_r\right)\frac{V}{m}$ in spherical coordinate, find the potential of point $(2\, m, \pi, \frac{\pi}{2})$ with respect to $(4\, m, 0, \pi)$.

$$V_{AB} = -\int_4^{2} \left(-\frac{16}{r^2}\right) dr = -4\ V$$