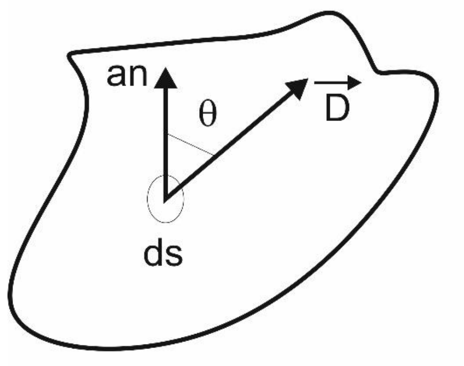
Electric Flux density $\overrightarrow{D}$.

Electric flux φ originates on positive charge and terminates on negative charge. In the absence of negative charge, the flux φ terminates at infinity.

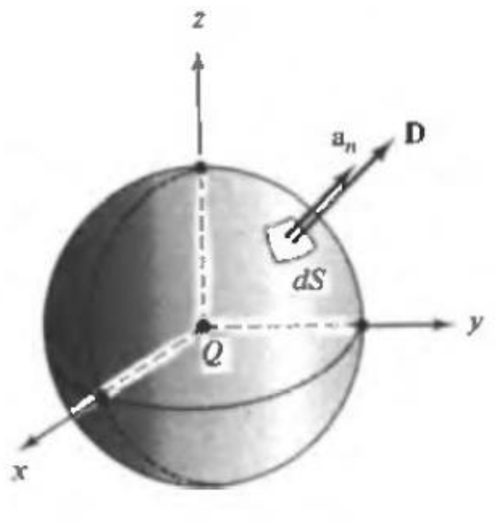
$$\varphi = Q$$

$$\vec{D} = \frac{d\varphi}{ds} \quad a_n \qquad C/m^2$$



Gauss law.

Total electric flux through closed surface is equal to the net charge within the surface.



$\oint \vec{D} \cdot \overrightarrow{ds} = Q$ gauss law

$Q = D \int ds$; $\int ds =$ Integral of the circle surfaces $4\pi r^2$

$$Q = D(4\pi r^2)$$

$$\therefore \vec{D} = \frac{Q}{4\pi r^2}$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$\therefore \vec{D} = \varepsilon_0 \vec{E} \quad free\ space \ ; \quad \vec{D} = \varepsilon \vec{E} \ \ in\ general$$

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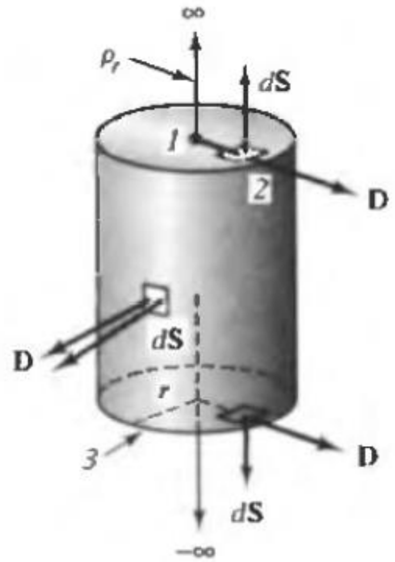
1. Use special guassian surface to find $\vec{D}$ due to uniform line charge ρ_ℓ C/m?



$$\oint D \cdot ds = Q$$

$$\int_1 D \cdot ds + \int_2 D \cdot ds + \int_3 D \cdot ds$$

$$= \int_2 D \cdot ds = D \int ds$$

$$Q = D(2\pi r L)$$

$$\therefore D = \frac{Q}{2\pi r L} \quad ; Q = \rho_\ell \, L$$

$$\therefore D = \frac{\rho_\ell \, L}{2\pi r L} = \frac{\rho_\ell}{2\pi r} \quad a_r$$

$$\vec{E} = \frac{\rho_\ell}{2\pi \varepsilon_0 r} \quad a_r$$

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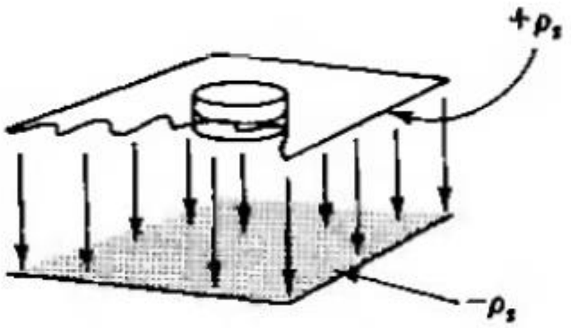
2. Capacitor charge on upper plate $+\rho_s \ \left(\frac{C}{m^2}\right)$ and lower plate $-\rho_s \ \left(\frac{C}{m^2}\right)$ neglect fringing and use Gauss law to find E and D?



$$\oint D \cdot ds = \int_{top} D \cdot ds + \int_{lower} D \cdot ds$$

$$+ \int_{bottom} D \cdot ds$$

$$Q = \int_{bottom} D \cdot ds$$

$$\rho_s A = D\int ds \qquad ; \quad \rho_s A = DA$$

$$\vec{D} = \rho_s \qquad ; \ \vec{E} = \frac{\rho_s}{\varepsilon_0}\ a_r$$

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3. Find the charge in the volume defined by $0 \le x \le 1$; $0 \le y \le 1$; $0 \le z \le 1$ if $\rho = 30\,x^2\,y\ (\mu c/m^3)$?

$$Q = \int \rho_v\, dv$$

$$Q = \int_0^1\int_0^1\int_0^1 30\,x^2\,y\ dx\,dy\,dz\ = 5\,\mu c$$

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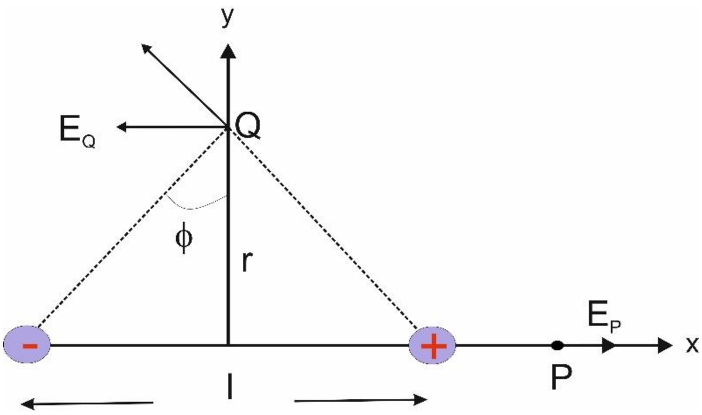
4. Find the net charge enclosed in a cube 2 m on an edge, parallel to the axes and centerd at the origin , if the charge density is $\rho = 50\,x^2\cos(\frac{\pi}{2}y)\ \ (\mu c/m^3)$? H.W
5. A closed surface S contains a finite line charge distribution $0 \le \ell \le \pi$ with charge density $\rho_\ell = -\rho_0\ \sin\frac{\ell}{2}\ \ (c/m)$,what net flux crosses the surface S? H.W

Field of dipole:

1. Electric field of equatorial on dipole

$$E_{q+} = \frac{1}{4\,\pi\varepsilon_0}\left(\frac{q}{r^2+\ell^2}\right)$$

$$E_{q-} = \frac{1}{4\,\pi\varepsilon_0}\left(\frac{q}{r^2+\ell^2}\right)$$



$E\sin\theta$ is in opposite direction ,it is cancelled

$E\cos\theta$ is in same direction , it is add

$$E_Q = 2\,E\cos\theta$$

$$E_Q = 2\frac{1}{4\,\pi\varepsilon_0}\left(\frac{q}{r^2+\ell^2}\cdot\frac{\ell}{(r^2+\ell^2)^{\frac{1}{2}}}\right)$$

$$if\ r \gg \ell \;\;;\;\; \ell^2 \cong 0$$

$$E_Q = 2\frac{1}{4\,\pi\varepsilon_0}\cdot\frac{q\ell}{r^3} \;\;;\;\; 2q\ell = P$$ Dipole moment

$$\overrightarrow{E_Q} = \frac{P}{4\,\pi\varepsilon_0 r^3}\;(-a_x)$$

2. Electric field at a point on the axis of a dipole

$$E_{q+} = \frac{1}{4\,\pi\varepsilon_0}\left(\frac{q}{(r-\ell)^2}\right)$$

$$E_{q-} = \frac{1}{4\,\pi\varepsilon_0}\left(\frac{q}{(r+\ell)^2}\right)$$

$$E_P = \frac{q}{4\,\pi\varepsilon_0}\left(\frac{1}{(r-\ell)^2} - \frac{1}{(r+\ell)^2}\right)$$

$$E_P = \frac{q}{4\,\pi\varepsilon_0}\left(\frac{(r+\ell)^2-(r+\ell)^2}{(r^2-\ell^2)^2}\right)$$

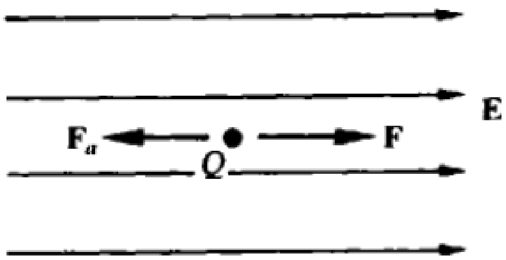
$$E_P = \frac{1}{4\,\pi\varepsilon_0}\cdot\frac{4q\,r\,\ell}{(r^2-\ell^2)^2}$$

$$if\ r \gg \ell \;\;;\;\; \ell^2 \cong 0$$

$$\overrightarrow{E_p} = \frac{2P}{4\,\pi\varepsilon_0 r^3}\;(a_x)$$

Energy and electric potential of charge system

A charge Q experiences a force F in an electric field E. In order to maintain the charge in equilibrium a force F_a must be applied in opposition.



$F = Q\,E\,;\qquad F_a = -QE$

Where the work is defined as a force acting over a distance. Therefore, differential amount of work dw is done when the applied force F_a produces as a differential displacement dl of the charge .

$dw = F_a \cdot dl = -QE \cdot dl$

Note:

When Q is positive and dl is in the direction, $dw = -QEdl \; < 0$ this mean the work was done by the electric field. On the other hand, when the work is positive this mean the work done against the electric field.

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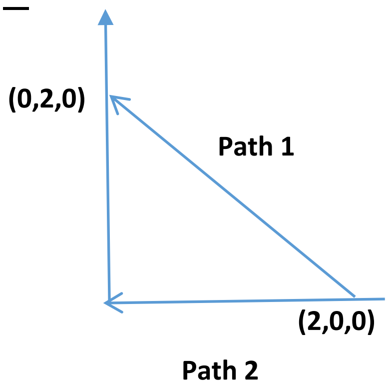
1. Find the work done in moving a charge of +2 C from (2,0,0) to (0,2,0) along straight line as in figure if $\vec{E} = 2x\,a_x - 4ya_y \;\; V/m$?



$dw = -QE.dl$

$dw = -2(2x\,a_x - 4ya_y)\;.(dx\,a_x + dy\,a_y + dz\,a_z)$

$dw = -4x\,dx + 8y\,dy$

$x + y = 2 \quad path\ equation$

$y = 2 - x \qquad dy = -dx$

$dw = -4x\,dx + 8(2 - x)\,(-dx)$

$dw = 4xdx - 16dx$

$$w = \int_{2}^{0} (4x - 16)dx = 24 \ J$$

For verification the result you can found w1 and w2, then w=w1+w2

It should be the same result.

Find $\oint E.dl$???? H.W

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Electric potential between two points

The potential of point A with respect to point B is define as the work done in moving a unit positive charge Q_u from B to A.

$$V_{AB} = \frac{W}{Q} = -\int_{B}^{A} E \cdot dl$$

1. Find the potential of A$(1, \emptyset, z)$ with respect to B $(3, \grave{\emptyset}, \grave{z})$ in cylindrical coordinate where electric field due to line charge on Z-axes is given by $\vec{E} = \frac{50}{r} a_r \ V/m$?

$$V_{AB} = -\int_{B}^{A} E \cdot dl \qquad ; \qquad V_{AB} = -\int_{B}^{A} E_r \cdot dr$$

$$= -\int_{1}^{3} \frac{50}{r} \, dr = -50 \ln r|_1^3 = -50 \ln \frac{1}{3} = 54.9 \, V$$

The work of one-point charge:

$$V_{AB} = \frac{W}{Q} = -\int_{B}^{A} E \cdot dl$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$V_{AB} = -\int \frac{Q}{4\pi\varepsilon_0 r^2}\, dl \quad ; dl = dr$$

$$V_{AB} = -\frac{Q}{4\pi\varepsilon_0}\int_{r_A}^{r_B} \frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right) \quad ; if\ r_B = \infty$$

$$V_{AB} = \frac{Q}{4\pi\varepsilon_0 r}$$

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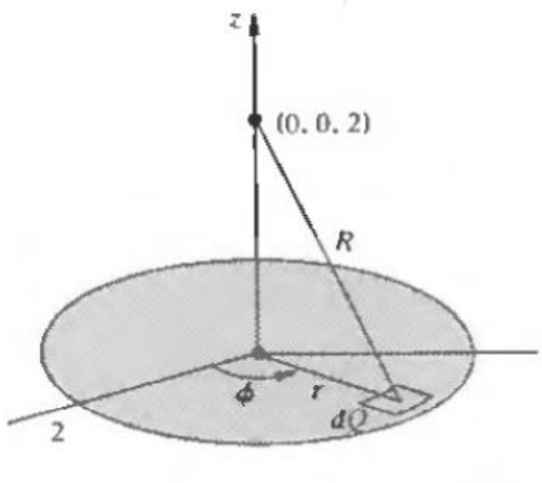
1. Charge $\left(\frac{40}{3}\right) nC$ is uniformly distributed around ring of radius (2 m) Find the potential at point on axis (2m) from the plane of ring?



$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

$$V = \int \frac{\rho_s\, ds}{4\pi\varepsilon_0 r}$$

$$\rho_s = \frac{Q}{A} = \frac{\frac{40}{3}\times 10^{-9}}{2\pi r} = \frac{10^{-8}}{3\pi}\ C/m$$

$$dQ = \rho_s\, ds \quad ; ds = r\, dr\, d\emptyset$$

$$R = \sqrt{r^2 + 4}$$

$$V = \frac{30}{\pi}\int_0^{2\pi}\int_0^{2} \frac{r\, dr\, d\emptyset}{\sqrt{r^2+4}} = 49.7\ V$$

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2. Given the field $E = \left(-\frac{16}{r^2}\, a_r\right)\frac{V}{m}$ in spherical coordinate, find the potential of point $(2\ m, \pi, \frac{\pi}{2})$ with respect to $(4\ m, 0, \pi)$.

$$V_{AB} = -\int_4^2 \left(-\frac{16}{r^2}\right) dr = -4\ V$$