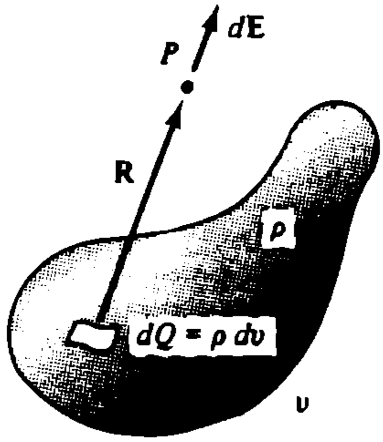
Charge Distribution

1.Volum charge

When charge is distributed throughout specified volume, each charge element contribute to the electric field at an external point. The charge density defined as:

$$\rho = \frac{dQ}{dv} \quad (\frac{C}{m^3})$$



Each differential charge dQ produces a differential electric field.

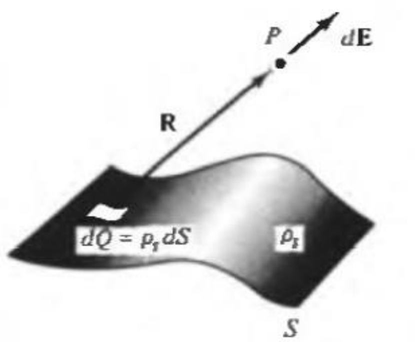
$$dE = \frac{dQ}{4\pi\varepsilon_0 R^2} a_R$$

Assuming that the only charge in the region is contained within the volume, the total electric field at P is obtained by integration over the volume:

$$E = \int \frac{\rho a_R}{4\pi\varepsilon_0 R^2} dv$$

2.Sheet charge

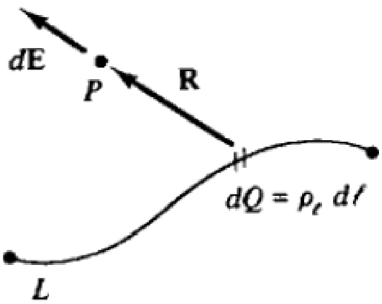
Charge may also be distributed over the surface or sheet. Then each differential dQ on the sheet results in a differential electric field. At a point P as in figure. If the surface charge density is ρ_s and if no other charge present in the region, then the total electric field on the point P is:



$$E = \int_s \frac{\rho_s a_R}{4\pi\varepsilon_0 R^2} ds$$

3.line charge

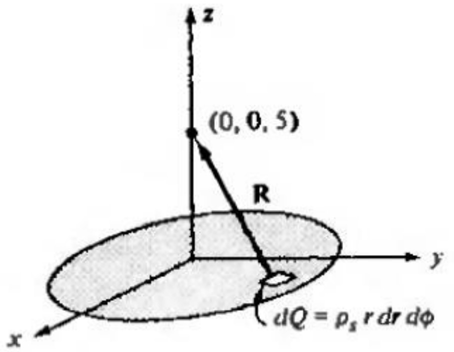
If charge distributed over a curved line, each differential charge dQ along the line produces a differential electric field . At P as in figure if the line charge density is ρ_ℓ and if no other charge present in the region, then the total electric field on the point P is:



$$E = \int_L \frac{\rho_\ell a_R}{4\pi\varepsilon_0 R^2}\, d\ell$$

Problems:

1. Find the force point charge of $50\ \mu c$ at (0,0,5) m due to charge of $500\pi\ \mu c$ that is uniformly distribution over circular disk $r \le 5\ m \quad z = 0\ m$?



$$\overrightarrow{dF} = \frac{(50\times 10^{-6})(\rho_s ds)}{4\pi(\frac{10^{-9}}{36\pi})}$$

$$\overrightarrow{dF} = \frac{(50\times 10^{-6})(\rho_s\, r\, dr\, d\emptyset)}{4\pi(\frac{10^{-9}}{36\pi})}$$

$$\rho_s = \frac{dq}{ds}$$

$$\rho_s = \frac{500\pi}{25\pi} = 20$$

$$\overrightarrow{dF} = \frac{(50\times 10^{-6})(20\times 10^{-6} r\, dr\, d\emptyset)}{4\pi(\frac{10^{-9}}{36\pi})}$$

$$\vec{r} = -ra_r + za_z$$

$$\vec{r} = -ra_r + 5a_z$$

$$|r| = \sqrt{r^2 + 25}$$

$$\vec{F} = \frac{(50\times10^{-6})(20\times10^{-6} r\,dr\,d\emptyset)}{4\pi(\frac{10^{-9}}{36\pi})\cdot(r^2+25)}\cdot\frac{-ra_r+5a_z}{\sqrt{r^2+25}}$$

$$\vec{F} = \int_0^{2\pi}\int_0^{5}\frac{(50\times10^{-6})(20\times10^{-6})\cdot 5r\,dr\,d\emptyset}{4\pi(\frac{10^{-9}}{36\pi})\cdot(r^2+25)^{3/2}}\,a_z$$

$$\vec{F} = 90\pi\int_0^{5}\frac{r\,dr}{(r^2+25)^{3/2}}\,a_z = 16.56\ a_z\ N$$

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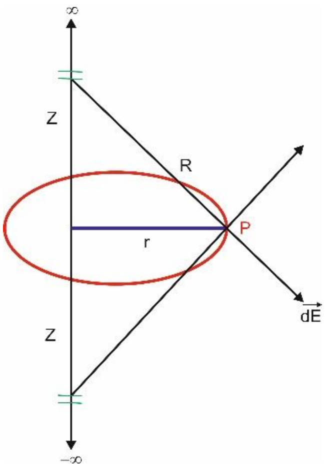
2.Charge is distributed uniformly along infinite straight line with density (ρ_ℓ) develop expression of (E) at general point (P)?



$r = ra_r - za_z\ ;\ |r| = \sqrt{r^2+z^2}$

$$\overrightarrow{dE} = \frac{dq}{4\pi\varepsilon_0 r^2}\,a_r$$

$$\overrightarrow{dE} = \frac{\rho_\ell dz}{4\pi\varepsilon_0(r^2+z^2)}\cdot\frac{ra_r - za_z}{\sqrt{r^2+z^2}}$$

$$\vec{E} = \int_{-\infty}^{\infty}\frac{\rho_\ell\, rdz}{4\pi\varepsilon_0(r^2+z^2)^{3/2}}\,a_r$$

$$\vec{E} = \frac{\rho_\ell}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{rdz}{(r^2+z^2)^{3/2}}\,a_r$$

$\vec{E} = \frac{\rho_\ell}{2\pi\varepsilon_0}a_r$ infinite line charge

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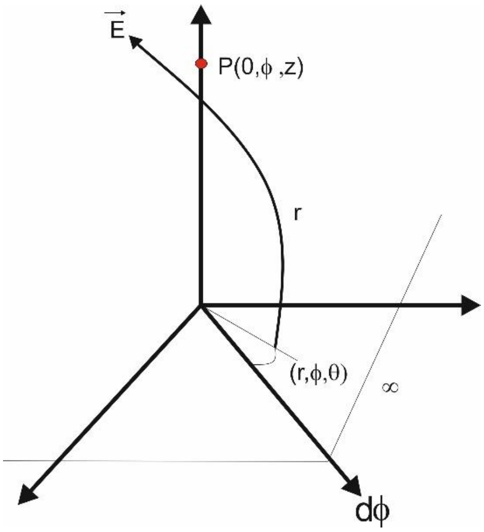
3.Develop expression for (E) due to charge uniformly distributed on infinite plane with densityρ_s?



$$r = -ra_r + za_z \ ; \ |r| = \sqrt{r^2 + z^2}$$

$$\overrightarrow{dE} = \frac{\rho_s \, r \, dr \, d\emptyset}{4\pi\varepsilon_0 (r^2 + z^2)} \cdot \frac{-ra_r + za_z}{\sqrt{r^2 + z^2}}$$

$$\vec{E} = \frac{\rho_s}{4\pi\varepsilon_0} \int_0^{2\pi} \int_0^{\infty} \frac{r \, z \, dr \, d\emptyset}{(r^2 + z^2)^{3/2}} \, a_z$$

$$\vec{E} = \frac{\rho_s}{2\varepsilon_0} a_z$$

infinite plane charge