Coulomb's Law

There is a force between two charges which is directly proportional to the charge magnitude and inversely proportional to the square of the separation distance. This is coulomb's law, which was developed from work with small charged bodies and delicate torsion balance. In vector form, it is stated thus:

$F = \frac{Q_1 Q_2}{4\pi\varepsilon r^2} a_{12} \quad ; \frac{\vec{r}}{|r|}$

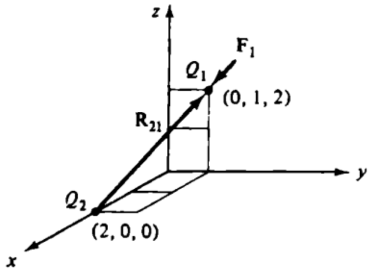
The force is in newton (N), the distance is in meter (m), and the (derived) unit of charge is in coulomb (C). ε is the permittivity of the medium, with the units (C^2/N.m) or equivalently, farad per meter (F/m). For free space in vacuum:

$$\varepsilon = \varepsilon_0 = 8.854 \times \frac{10^{-12} F}{m} \approx \frac{10^{-9}}{36\pi}\ F/m$$

For media other than free space $\varepsilon = \varepsilon_0 \varepsilon_r$ where ε_r is relative permittivity or dielectric constant.

Problems:

1. Find the force $Q_1 = 20\ \mu c$ due to charge $Q_2 = -300\ \mu c$ where Q1 at (0,1,2) m and Q2 at (2,0,0) m?



$\vec{r} = -2a_x + a_y + 2a_z$

$|r| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (Z_2 - Z_1)^2}$

$|r| = 3$

$$\vec{F} = \frac{(20 \times 10^{-6})(-300 \times 10^{-6})}{4\pi\left(\frac{10^{-9}}{36\pi}\right).9} \cdot \frac{-2a_x + a_y + 2a_z}{3}$$

$\vec{F} = 6\left(\frac{2a_x - a_y - 2a_z}{3}\right)$

$\vec{F} = 4a_x - 2a_y - 4a_z \quad N$

The magnitude is 6 N and the direction is such as Q1 attracted to Q2 (unlike charged attract).

****

2.Two point charges, $Q_1 = 50\ \mu c$ and $Q_2 = 10\ \mu c$ located at (-1,1,-3)m and (3,1,0)m ,respectively, Find the force Q_1?

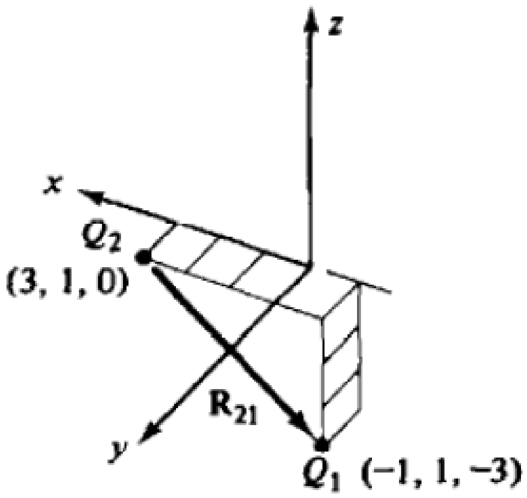
$$\vec{R}_{21} = -4a_x - 3a_z$$

$$a_{21} = \frac{-4a_x - 3a_z}{5}$$

$$\vec{F_t} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{21}} a_{21}$$

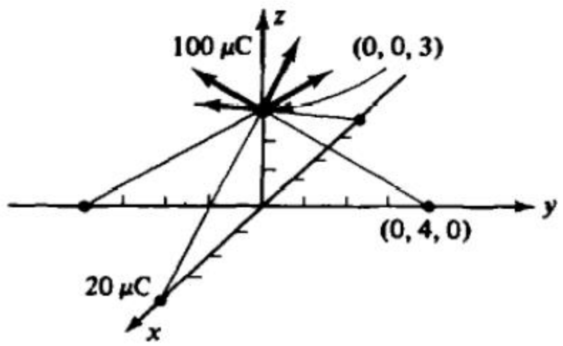
$$= \frac{(50 \times 10^{-6})(10^{-5})}{4\pi\left(\frac{10^{-9}}{36\pi}\right)(5)^2} \cdot \frac{-4a_x - 3a_z}{5}$$

$$= (0.18)(-0.8a_x - 0.6a_y)N$$



****

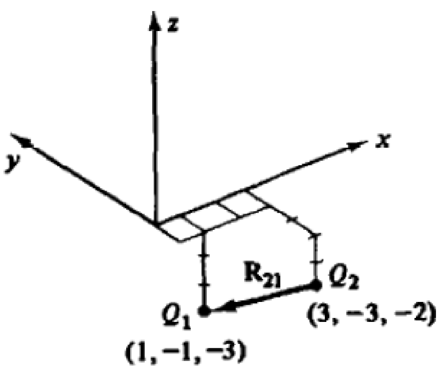
3.Find The force on a 100 μc charge at (0,0,3) m, if four like charges of 20 μc are located on X and Y axes at $\pm$4m?H.W



4.Point charge $Q1 = 300\ \mu c$ located at (1,-1,-3)m experiences a force $F = 8\ a_x - 8a_y + 4a_z\ \ N$ due to point charge $Q2$ at (3,-3,-2)m, Determine $Q2$?

$$\vec{R}_{21} = -2a_x + 2a_y - a_z$$

$$\vec{F_t} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{21}} a_{21}$$



$$8\,a_x - 8a_y + 4a_z = \frac{(300\times 10^{-6})Q_2}{4\pi\left(\frac{10^{-9}}{36\pi}\right)(3)^2}\cdot\frac{-2a_x + 2a_y - a_z}{3}$$

$$Q_2 = -40\,\mu c$$

****

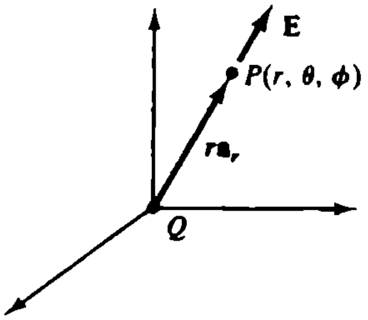
Electric Field intensity ($\vec{E}$):

The electric field intensity E due to Q is defined to be the force for unit charge on Q_t:

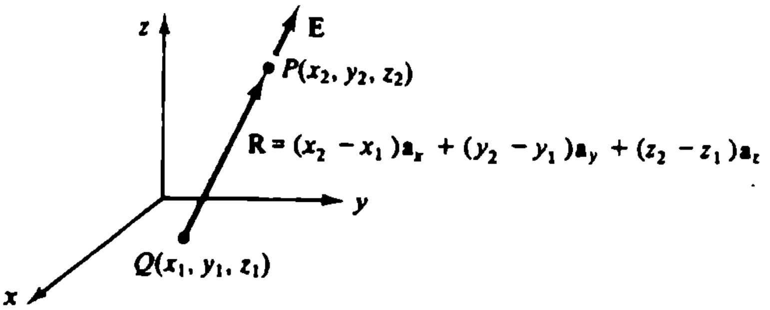
$$\vec{E} = \frac{\vec{F_t}}{Q_t}$$

For Q at the origin point of spherical coordinate system, the electric field at an arbitrary point P is figure (a):

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\,a_r$$



(b) Spherical



(a)Cartesian

In an arbitrary Cartesian coordinate system figure (b):

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\,a_R$$

The unit of E is newton per coulomb (N/C) or equivalent volts per meter (V/m).

Example: Find (E) at (0,3,4) m in Cartesian coordinate due to a point charge $Q = 0.5\ \mu c$ at the origin point.

In the case

$$\vec{R} = 3a_y + 4a_z \qquad R = 5 \qquad a_R = 0.6\ a_y +\ 0.8\ a_z$$

$$\vec{E} = \frac{0.5 \times 10^{-6}}{4\pi(\left(\frac{10^{-9}}{36\pi}\right)(5)^2} 0.6\ a_y +\ 0.8\ a_z$$

$$|E| = 180 \frac{V}{m} \text{ in the direction } 0.6\ a_y +\ 0.8\ a_z$$

****

Example: Find (E) at the origin point due to a point charge of 64.4C located at (-4,3,2) m in Cartesian coordinate.

$$|R| = \sqrt{29}$$

$$\vec{R} = 4a_x - 3a_y - 2a_z$$

$$\vec{E} = \frac{64.4 \times 10^{-9}}{4\pi(\left(\frac{10^{-9}}{36\pi}\right)(29)} \cdot \frac{4a_y - 3a_y - 2a_z}{\sqrt{29}}$$

$$\vec{E} = (20.0) \frac{4a_y - 3a_y - 2a_z}{\sqrt{29}} \quad V/$$

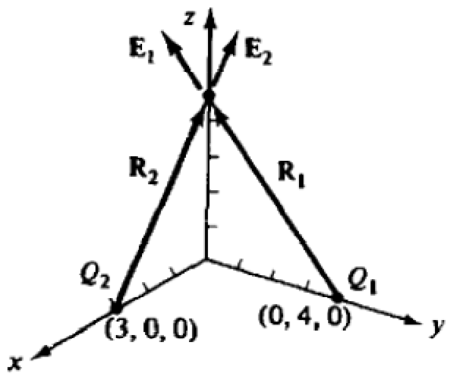
****

Example: Find (E) at (0,0,5) m due to $Q_1 = 0.35\ \mu c$ at (0,4,0)m and $Q_2 = -0.55\ \mu c$ at (3,0,0)m as shown in figure?

$$\vec{R}_1 = -4a_y + 5a_z$$

$$\vec{R}_2 = -3a_x + 5a_z$$

$$\vec{E}_1 = \frac{0.35\times10^{-6}}{4\pi(\left(\frac{10^{-9}}{36\pi}\right)(41)} \cdot \frac{-4a_y + 5a_z}{\sqrt{41}}$$



$$\vec{E}_1 = -48a_y + 60a_z \ \ V/m$$

$$\vec{E}_2 = \frac{-0.55\times10^{-6}}{4\pi(\left(\frac{10^{-9}}{36\pi}\right)(34)} \cdot \frac{-3a_x + 5a_z}{\sqrt{34}}$$

$$\vec{E}_1 = 74.9a_x + 124.9a_z \ \ V/m$$

$$\vec{E}_t = 74.9a_x - 48a_y - 64.9a_z \ \ V/m$$

****

Example.Find the expression for the electric field at point P due to point charge Q at (x_1,y_1,z_1). Repeat with the charge placed at origin?H.W