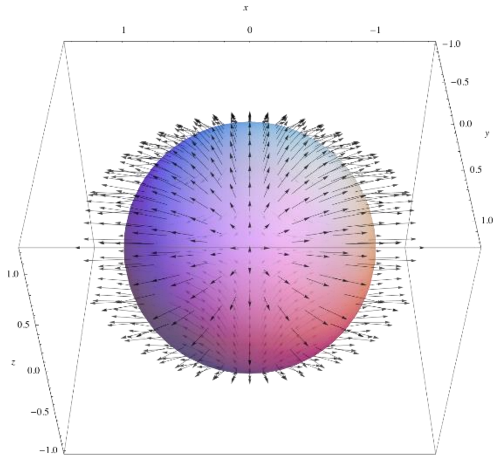
The Divergence Theorem

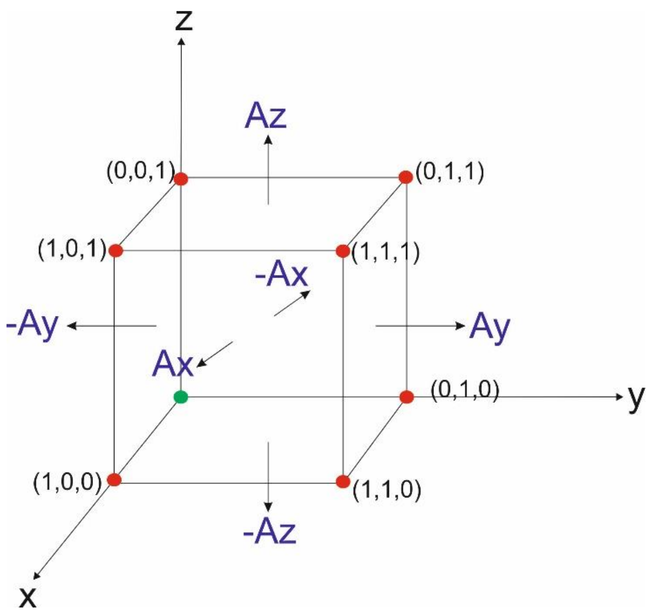
The integral of normal component of vector field over closed surface yield some result as the integral of the divergence of vector field throughout the volume enclosed the surface.

$$\oint \vec{F} \cdot \overrightarrow{ds} = \int_V (\vec{\nabla} \cdot \vec{F}) dv$$



Example:

1. Consider the vector field given by $\vec{A} = 2xy\, a_x + 3a_y + Z^2 y a_z$ Suppose t=he surface is cube with side of unity area as in figure verify the divergence theorem for this vector field.



$$\oint \vec{A} \cdot \overrightarrow{ds} = \int_V (\vec{\nabla} \cdot \vec{A}) dv$$

$$\vec{\nabla} \cdot \vec{A} = 2y + 2zy$$

$$\therefore \int \vec{\nabla} \cdot \vec{A} = \int_0^1 \int_0^1 \int_0^1 (2y + 2zy)\, dxdydz$$

$$\therefore \int \vec{\nabla} \cdot \vec{A} = \frac{3}{2}$$

$$\oint \vec{A} \cdot \overrightarrow{ds} = \int_0^1 \int_0^1 Az_{z=1}\, dxdy + \int_0^1 \int_0^1 -Az_{z=0}\, dxdy + \int_0^1 \int_0^1 Ay_{y=1}\, dxdz + \int_0^1 \int_0^1 -Ay_{y=0}\, dxdz + \int_0^1 \int_0^1 Ax_{x=1}\, dydz + \int_0^1 \int_0^1 -Ax_{x=0}\, dydz$$

$$= \int_0^1 \int_0^1 Z^2 y\, dxdy + \int_0^1 \int_0^1 -Z^2 y\, dxdy + \int_0^1 \int_0^1 3\, dxdz + \int_0^1 \int_0^1 -3\, dxdz + \int_0^1 \int_0^1 (2xy)\, dydz + \int_0^1 \int_0^1 (-2xy) dy\, dz$$

$$= \frac{1}{2} + 0 + 1 = \frac{3}{2}$$

2. Verify divergence theorem of vector field $\vec{F} = x\, a_x - 2ya_y$ when the closed surface is box with corners (2,1,0),(2,0,1),(2,1,1), (2,0,0),(0,1,1),(0,0,0),(0,0,1)? H.W

3. Given that $\vec{F} = x\, a_x - 2ya_y$ Evaluate both sides of divergence theorem of the volume enclosed by the shell r=2? H.W

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