

Differentiation with vectors and matrices

Assume that $f(\underline{x})$ be continuous function of the element of $\underline{x}' = (x_1, x_2, \dots, x_n)$ whose first and second partial derivative exists

$$\frac{\partial f(\underline{x})}{\partial x_i}, \quad \frac{\partial^2 f(\underline{x})}{\partial x_i \partial x_j} \quad \forall i, j = 1, 2, \dots, n$$

Where $f(\underline{x})$ is function of vector \underline{x}

$$\frac{\partial f(\underline{x})}{\partial \underline{x}} = \begin{bmatrix} \frac{\partial f(\underline{x})}{\partial x_1} \\ \frac{\partial f(\underline{x})}{\partial x_2} \\ \vdots \\ \frac{\partial f(\underline{x})}{\partial x_n} \end{bmatrix}$$

$$(1) f(\underline{x}) = \underline{a}' = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \Rightarrow \frac{\partial f(\underline{x})}{\partial \underline{x}} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \underline{0}$$

$$(2) f(\underline{x}) = \underline{a}'\underline{x} = \underline{x}'\underline{a} = \sum a_i x_i \quad \text{where } \underline{a} \text{ is a vector of constant}$$

$$\frac{\partial f(\underline{x})}{\partial \underline{x}} = \underline{a}$$

or \underline{a} is a column vector