Quadratic forms

الاشكال التربيعية

A function $f(x_1, x_2, \dots, x_n)$ of a real variables $x_1, x_2, \dots, x_n$ is define to be quadratic forms of

$$f(x_1, x_2, \dots, x_n) = a_{11}X_1^2 + a_{22}X_2^2 + \dots + a_{nn}X_n^2 +$$
$$2a_{12}X_1X_2 + 2a_{13}X_1X_3 + \dots + 2a_{1n}X_1X_n + \dots$$
$$\cancel{2a_{21}X_2X_1 + 2a_{23}X_2X_3} + \dots + 2a_{(n-1)n}X_{(n-1)}X_n$$
$$= \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}X_iX_j = \underline{X}'A\underline{X} \rightarrow \text{scalar}$$

the symm. matrix A is defined to be the matrix of (Q.F) where $A = [a_{ij}]$ $i, j = 1, 2, \dots, n$

Note: the quadratic form $Q = \underline{Y}' A \underline{Y}$ is called positive definite if $q > 0$ for all elements in $\underline{Y}$ and is semi positive definite if $q \geq 0$ $\forall$ element in $\underline{Y}$, $q = \sum_i \underbrace{d_i}_{\text{عناصر القطر}} Y_i^2$

the forms matrix	هي التي تحدد نوع الشكل التربيعي type of the form	d_i Positive	Zero	Negative
non-Singular	p.d.	✓		
Singular	S.p.d.	✓	✓	
non Singular	غير محددة i.d.	✓		✓
Singular	i.d.		✓	
non Singular	N.d.			✓
Singular	S.N.d		✓	✓
Non Singular	Null عدم وجود شكل تربيعي		✓	

theorem

(I) A is p.d. matrix iff all (leading principle minor) determinants are positive
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(II) A is s.p.d. matrix iff the first r leading principle minor determinants are positive and the remaining $n-r$ are zero, then $r(A) = r$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

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$P_1 = a_{11}$, $P_2 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

$P_3 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ & a_{22} & a_{23} \\ \text{Symm.} & & a_{33} \end{vmatrix}$