which have the properties $c_i'c_i = 1$ for all i and

AHS
اذا كانت مصفوفة ال trans على الثاني CC'
واذا لاحقت التحويرمنه فا $C'C$

$c_i'c_j = 0$ for all $i \neq j$ we have $C'C = I_p = CC'$

that is if C satisfies $C'C = I_p$, it necessarily

$\boxed{CC' = I_p}$, from which we see that rows of C are also

normalized and mutually orthogonal (متعامدة بشكل متبادل)

It is clear from $C'C = CC' = I_p$ that $C^{-1} = C'$

for an orthogonal matrix C. (orthonormal matrix)

Note: If C is an orthogonal matrix, then

the diagonal elements of C satisfies

$$-1 \leq c_{ii} \leq 1 \quad \text{and} \quad |C| = \pm 1$$

Example: If $\underline{X} = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix}$ and $\underline{Y} = \begin{bmatrix} 1/3 \\ 2/3 \\ 2/3 \end{bmatrix}$

Are $\underline{X}$ and $\underline{Y}$ orthogonal, normalized and mutually orthogonal (orthonormal)

Solution: $\underline{Y}'\underline{X} = \left(\frac{1}{3} \ \frac{2}{3} \ \frac{2}{3}\right)\begin{pmatrix} 0 \\ 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix} = \frac{\sqrt{2}}{3} - \frac{\sqrt{2}}{3} = 0$

so they are orthogonal

$$\left.\begin{aligned} \underline{X}'\underline{X} &= \left(0 \ \frac{1}{\sqrt{2}} \ \frac{-1}{\sqrt{2}}\right)\begin{pmatrix} 0 \\ 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix} = 0 + \frac{1}{2} + \frac{1}{2} = 1 \\ \underline{Y}'\underline{Y} &= \left(\frac{1}{3} \ \frac{2}{3} \ \frac{2}{3}\right)\begin{pmatrix} \frac{1}{3} \\ 2/3 \\ 2/3 \end{pmatrix} = \frac{1}{9} + \frac{4}{9} + \frac{4}{9} = \frac{9}{9} = 1 \end{aligned}\right\} \text{normalized}$$

$\because \underline{Y}'\underline{X} = 0$ and $\underline{X}'\underline{X} = 1$, $\underline{Y}'\underline{Y} = 1 \Longrightarrow \underline{X}$ and $\underline{Y}$ are mutually orthogonal or (orthonormal)