Orthogonal Vectors and Matrices

two vectors $\underline{X}$ and $\underline{Y}$ of the same size are said to be orthogonal iff

$$\underline{X}'\underline{Y} = x_1 y_1 + x_2 y_2 + \cdots + x_n y_n = 0 \quad , \text{ that is}$$

if the inner product of the vectors will be zero

$\cos(\theta) = 0 \quad , \theta = 90$ ومعناها ان المتجهات بزاوية ٩٠ (متعامدة)

$$\cos(\theta) = \frac{\underline{X}'\underline{Y}}{\|\underline{X}\|\,\|\underline{Y}\|}$$

قياسي

If $\underline{X}'\underline{X} = 1$, the vector $\underline{X}$ is said to be normalized.

the vector $\underline{X}$ can always be normalized by dividing by it's length, $\sqrt{\underline{X}'\underline{X}}$. thus $\underline{c} = \dfrac{\underline{X}}{\sqrt{\underline{X}'\underline{X}}}$ is normalized so that $\underline{c}'\underline{c} = 1$

* اي vector عندما نقسمه على طوله يتحول الى normalized vector (متجه قياسي)

A matrix $\mathbf{C} = (\underline{c}_1 \ \underline{c}_2 \cdots \underline{c}_p)$ whose columns are normalized and mutually orthogonal is called an orthogonal matrix

$$C'C = \begin{bmatrix} \underline{c}_1' \\ \underline{c}_2' \\ \vdots \\ \underline{c}_p' \end{bmatrix}_{p\times 1} (\underline{c}_1 \ \underline{c}_2 \cdots \underline{c}_p)_{1\times p} = \begin{bmatrix} \underline{c}_1'\underline{c}_1 & \underline{c}_1'\underline{c}_2 \cdots \underline{c}_1'\underline{c}_p \\ \underline{c}_2'\underline{c}_1 & \underline{c}_2'\underline{c}_2 \cdots \underline{c}_2'\underline{c}_p \\ \vdots & \vdots \\ \underline{c}_p'\underline{c}_1 & \underline{c}_p'\underline{c}_2 \cdots \underline{c}_p'\underline{c}_p \end{bmatrix}_{p\times p}$$