

linear transformations التحويلات الخطية

$$\underline{y} = A \underline{x} + \underline{b} \quad \text{If } A \text{ is a nonsingular}$$

$$A \underline{x} = \underline{y} - \underline{b} \Rightarrow \underline{x} = A^{-1}(\underline{y} - \underline{b})$$

Trace of matrix

the sum of the diagonal elements of the square matrix. For $n \times n$ matrix

$$A, \text{ trace } A = \text{tr}(A) = \sum_{i=1}^n a_{ii} \\ = a_{11} + a_{22} + \dots + a_{nn}$$

Some Properties

$$① \text{tr}(A+B) = \text{tr}(A) + \text{tr}(B)$$

$$② \text{tr}(AB) = \text{tr}(BA), \text{ it's not necessary } AB=BA \text{ or } A \text{ and } B \text{ are square matrices}$$

$$③ \text{tr}(A) = \text{tr}(A^{-1})$$

Example Let $A = \begin{pmatrix} 1 & 3 \\ 2 & -1 \\ 4 & 6 \end{pmatrix}, B = \begin{pmatrix} 3 & -2 & 1 \\ 2 & 4 & 5 \end{pmatrix}$, then

$$BA = \begin{pmatrix} 3 & 17 \\ 30 & 32 \end{pmatrix}$$

$$\text{tr}(BA) = 3 + 32 = 35$$

$$\text{H.W. } \text{tr}(AB)? \quad AB = \begin{pmatrix} 9 & 10 & 16 \\ 4 & -8 & -3 \\ 24 & 16 & 34 \end{pmatrix}$$

$$\text{tr}(AB) = 9 - 8 + 34 = 35$$