

المتجهات المتعامدة
orthogonal Projection

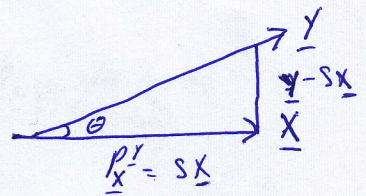
Orthogonal Projection of \underline{y} on \underline{x} denoted by $P_{\underline{x}}^{\underline{y}}$

نتيجة
vector

$$P_{\underline{x}}^{\underline{y}} = \frac{(\underline{y}, \underline{x}) \underline{x}}{\|\underline{x}\|^2} = \frac{(\underline{y}, \underline{x}) \underline{x}}{(\underline{x}, \underline{x})^{\frac{1}{2}}^2} = \frac{(\underline{y}, \underline{x}) \underline{x}}{(\underline{x}, \underline{x})}$$

If $S = \frac{(\underline{y}, \underline{x})}{(\underline{x}, \underline{x})}$, then $P_{\underline{x}}^{\underline{y}} = S \underline{x}$

$$\therefore S = \frac{(\underline{y}, \underline{x})}{\|\underline{x}\|^2}$$



$$(S \underline{x}, \underline{y} - S \underline{x}) = 0 \implies S(\underline{x}, \underline{y}) - S^2(\underline{x}, \underline{x}) = 0$$

to find θ

$$\cos(\theta) = \frac{\underline{x} \cdot \underline{y}}{\|\underline{x}\| \|\underline{y}\|} \implies (\underline{x}, \underline{y}) = \|\underline{x}\| \|\underline{y}\| \cos(\theta)$$

θ be angle between 2-vectors

$$\therefore S = \frac{(\underline{x}, \underline{y})}{(\underline{x}, \underline{x})} = \frac{\underline{x} \cdot \underline{y}}{\underline{x} \cdot \underline{x}} \implies P_{\underline{x}}^{\underline{y}} = \frac{\underline{x} \cdot \underline{y}}{\underline{x} \cdot \underline{x}} \cdot \underline{x}$$

example If $\underline{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\underline{y} = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$. find $P_{\underline{x}}^{\underline{y}}$

Solution

$$P_{\underline{x}}^{\underline{y}} = S \underline{x}, \quad S = \frac{(\underline{x}, \underline{y})}{(\underline{x}, \underline{x})} = \frac{\sum_{i=1}^3 x_i y_i}{\sum_{i=1}^3 x_i^2} = \frac{7}{3}$$

$$\therefore P_{\underline{x}}^{\underline{y}} = \frac{7}{3} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 7/3 \\ 7/3 \\ 7/3 \end{bmatrix}$$