

Mathematical Concepts

The Rank of a matrix A with order $(m \times n)$ is said to be $r(A)$ if the size of the largest nonsingular square submatrix of A is r and the matrix is said to be of rank zero iff all its elements are zero.

Some Properties of Rank

$$(1) r(A) = r(A^T) = r(A^T A) = r(AA^T)$$

$$(2) r(AB) = \min\{r(A), r(B)\}$$

$$(3) |A_{n \times n}| = 0 \text{ iff } r(A) < n$$

A square matrix is called singular if its determinant is zero. If the determinant is non zero the matrix is called nonsingular or full rank.

Example Find the rank of $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

Solution $|A| = \begin{vmatrix} 1 & 2 & 3 & | & 1 & 2 \\ 4 & 5 & 6 & | & 4 & 5 \\ 7 & 8 & 9 & | & 7 & 8 \end{vmatrix} = (45 + 84 + 96) - (105 + 48 + 72)$
 $= 225 - 225 = 0$

Row number two and column number three are removed

$$\begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix} = 8 - 14 = -6 \text{ non zero determinant}$$

$\therefore r(A) = 2$

