

Characteristic Roots and vectors

Eigen values and Eigen vectors

The characteristic roots of the $P \times P$ matrix A are the solutions for the determinant equation $|A - \lambda I| = 0$.
The determinant is P -th degree polynomial in λ .

A scalar λ is called eigen value of A if their exist

Some Properties of eigen values

- ① $\prod_{i=1}^P \lambda_i = |A|$
- ② $\sum_{i=1}^P \lambda_i = \text{tr}(A)$
- ③ The eigen values of a symmetric matrix A with real elements are all real.
- ④ The eigen values of a p.d. matrix are all positive.
- ⑤ The eigen values of a s.p.d. matrix of rank r are contains exactly r positive eigen values and the remaining are zero.
- ⑥ Each ^{eigen} value is associated with an eigen vector.
 $A \underline{x}_i = \lambda_i \underline{x}_i$, thus \underline{x}_i is defined to be characteristic vector of the $n \times n$ matrix A corresponding to the root λ_i .