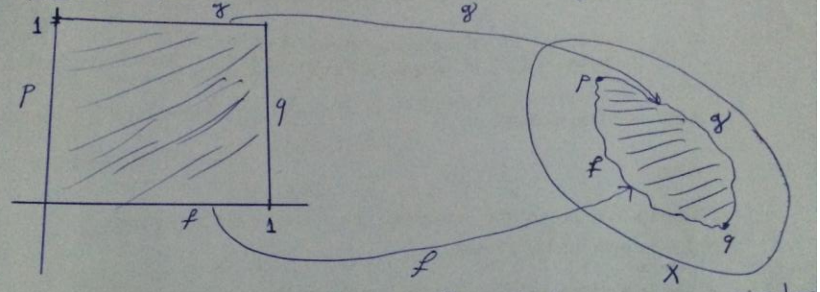
...c paths:

Let $f: I \to X$ and $g: I \to X$ be two paths with the same initial point $p \in X$ and the same terminal point $q \in X$. Then f is said to be homotopic to g, written $f \simeq g$, if there exists a continuous function $H: I^2 \to X$

such that $H(s,0) = f(s)$ $\quad H(0,t) = p$

$H(s,1) = g(s)$ $\quad H(1,t) = q$

And the function H is called a homotopy from f to g.



Example 10.7: Let X be the set of points between two circles as following diagrams

Diagram 1

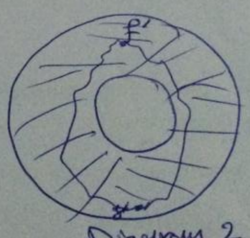


Diagram 2

Then the paths f and g in the diagram on the left (1) ... homotopic, whereas the paths f' and g' in the diagram (...

Example 10.8: Let $f: I \to X$ be any path. Then $f \simeq f$, i.e. f is homotopic to itself. In fact we can define a continu[ous] function $H: I^2 \to X$ as following
$H(s,t) = f(s)$ is homotopy from f to f.

Example 10.9: Let $f \simeq g$ and, say, $H: I^2 \longrightarrow X$ is homotopy from f to g. Then the function $\hat{H}: I^2 \to X$ defined by $\hat{H}(s,t) = H(s, 1-t)$ is homotopy from g to f and so $g \simeq f$.

Example 10.10: Let $f \simeq g$ and $g \simeq h$; say, $F: I^2 \to X$ is a homotopy from f to g and $G: I^2 \to X$ is a homotopy from g to h. The function $H: I^2 \to X$ defined by

$$H(s,t) = \begin{cases} F(s,2t) & \text{if } 0 \leq t \leq \frac{1}{2} \\ G(s, 2t-1) & \frac{1}{2} \leq t \leq 1 \end{cases}$$

is a homotopy from f to h, and so $f \simeq h$.