0.1: Before propos... and let B be a

et $G \cup H$ be a disconnection of A and let B be a connected subset of A. Then either $B \cap H = \phi$ or $B \cap G = \phi$ and so either $B \subset G$ or $B \subset H$.

proof:

Now $B \subset A$ and so $G \cap H \subset A^c \Rightarrow G \cap H \subset B^c$

$A \subset G \cup H \Rightarrow B \subset G \cup H$ and

Thus if both $B \cap G$ and $B \cap H$ are nonempty, then $G \cup H$ forms a disconnection of B. But B is connected hence the conclusion follows

Lemma 10.2: If A and B are connected sets which are not separated, then $A \cup B$ is connected.

proof: suppose $A \cup B$ is disconnected and suppose $G \cup H$ is disconnection of $A \cup B$. Since A is connected subset of $A \cup B$, either $A \subset G$ or $A \subset H$ (By lemma 10.1).

~~Since~~ Similarty, either $B \subset G$ or $B \subset H$.

Now, if $A \subset G$ and $B \subset H$ (or $B \subset G$ and $A \subset H$) (since $G \cup H$ is disconnetion for $A \cup B$)

$(A \cup B) \cap G = A$ and $(A \cup B) \cap H = B$

are separated sets. But this contradication

$\Rightarrow A \cup B$ is connected.

$((A \cup B) \cap G) \cap ((A \cup B) \cap H) = A \cap B = \phi$ since $G \cup H$ is disconnection of $A \cup B$

and if p is accumulation of $(A \cup B) \cap G$ and $p \in (A \cup B)$

$\Rightarrow H$ contain anther point of of $(A \cup B) \cap G$ since H is open $\Rightarrow ((A \cup B) \cap G) \cap ((A \cup B) \cap H) \neq \phi$

..., is path and arcwise is connected set ?

Proposition 10.1: A path is connected set

Proof:

Let $f: I \to X$ be any path in top. space X. $I = [0,1]$ in usual topology is connected set. Then by theorem 9.10. $f(I)$ is connected

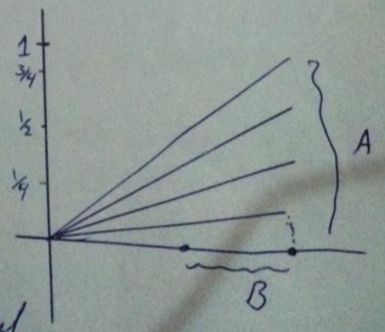
Theorem 10.2: Every arcwise connected set A is connected

Proof. If A is empty, then A is connected. suppose A is not empty; say, $p \in A$. Now A is arcwise connected and so, for each $a \in A$, there is a path $f_a : I \to A$ from p to a. Furthermore, $a \in f_a[I] \subset A$ and so $A = \cup\{f_a[I] : a \in A\}$. But $p \in f_a[I]$, for every $a \in A$; hence $\cap\{f_a[I] : a \in A\}$ is non-empty. Moreover, each $f_a[I]$ is connected, and A is connected. ~~If A is disconnected then we can divide A to parts G and H s.t. A = G∪H and G∩H = ∅ → c! since~~ $\cancel{f_a[I] \cap f'_a[I] \neq \emptyset \Rightarrow}$ ~~A is connected~~. by propo. 10.2.

Example 10.4: consider the following subsets of the plane $\mathbb{R}^2$

$A = \{\langle x, y\rangle : 0 \leq x \leq 1, y = x/n, n \in N\}$

$B = \{\langle x, 0\rangle : \frac{1}{2} \leq x \leq 1\}$



Here A consists of the points on the line segments joining the origin $\langle 0,0\rangle$ to the points $\langle 1, \frac{1}{n}\rangle$, $n \in N$ and B consists of the points on the x-axis between $\frac{1}{2}$ and 1. Now A and B are both arcwise connected