...ion 10.1: Before theorem 10.2

...t $\mathcal{A} = \{A_i\}$ be a class of connected subsets of X such that no two members of $\mathcal{A}$ are separated. Then $B = \bigcup_i A_i$ is connected.

proof: suppose B is not connected and $G \cup H$ is a disconnection of B. Now each $A_i \in \mathcal{A}$ is connected and so (Lemma 10.1) is contained in either G or H and disjoint from the other. Furthermore, any two members $A_{i_1}, A_{i_2} \in \mathcal{A}$ are not separated and so, by (lemma 10.2) $A_{i_1} \cup A_{i_2}$ is connected. Then $A_{i_1} \cup A_{i_2}$ is contained in G or H and ~~dis~~ disjoint from other $\Rightarrow \forall A_i \in \mathcal{A}$ and hence $B = \bigcup A_i$ $\subset G$ or $H \Rightarrow C!$ since $G \cup H$ is disconnection of $B \Rightarrow B$ is connected.

...roposition 10.2: Let $\mathcal{A} = \{A_i\}$ be a class of connected ...ubset of X with a nonempty intersection. Then ... $= \bigcup_i A_i$ is connected.

...oof: since $\bigcap A_i \neq \phi$ any two of $\mathcal{A}$ are not disjoint ...d so are not separated; hence by proposition 10.1 ... $= \bigcup_i A_i$ is connected.

...0.1: Before proposition ...
...et $G \cup H$ be a disconnection of A and let B be a connected subset of A. Then either $B \cap H = \phi$ or $B \cap G = \phi$ and so either $B \subset G$ or $B \subset H$.

proof:

Now $B \subset A$ and so $G \cap H \subset A^c \Rightarrow G \cap H \subset B^c$

$A \subset G \cup H \Rightarrow B \subset G \cup H$ and

Thus if both $B \cap G$ and $B \cap H$ are nonempty, then $G \cup H$ forms a disconnection of B. But B is connected hence the conclusion follows

Lemma 10.2: If A and B are connected sets which are not separated, then $A \cup B$ is connected.

proof: suppose $A \cup B$ is disconnected and suppose $G \cup H$ is disconnection of $A \cup B$. Since A is connected subset of $A \cup B$, either $A \subset G$ or $A \subset H$ (By lemma 10.1).

~~Since~~ Similarly, either $B \subset G$ or $B \subset H$.

Now, if $A \subset G$ and $B \subset H$ (or $B \subset G$ and $A \subset H$) (since $G \cup H$ is disconnetion for $A \cup B$)

$(A \cup B) \cap G = A$ and $(A \cup B) \cap H = B$ are separated sets. But this contradication

$\Rightarrow A \cup B$ is connected.

$((A \cup B) \cap G) \cap ((A \cup B) \cap H) = A \cap B = \phi$ since $G \cup H$ is disconnection of $A \cup B$

and if p is accumulation of $(A \cup B) \cap G$ and $p \in (A \cup B)$

$\Rightarrow H$ contain anther point of of $(A \cup B) \cap G$ since H is open $\Rightarrow ((A \cup B) \cap G) \cap ((A \cup B) \cap H) \neq \phi$