ample 8.10: Consider the following topology on $X=\{a,b,c,d,e\}$

$T=\{X,\emptyset,\{a\},\{c,d\},\{a,c,d\},\{b,c,d,e\}\}$

Now, X is disconnected since $\{a\}$ is open and closed

Example 8.11: The real line with usual top. is connected since $\mathbb{R}$ and $\emptyset$ are the only subsets of $\mathbb{R}$ which are both open and closed.

Theorem 8.10: Continuous image of connected space is connected

proof:

Let $f: X \to Y$ s.t X is connected < T.p $f(X)$ is connected >

Let $f(X)$ is disconnected $\Rightarrow G, H \in T_{f(X)}$ s.t

$G \cup H = f(X)$ and so $X = f^{-1}(G) \cup f^{-1}(H)$ and so

$G \cap H = \emptyset$ $\qquad \emptyset = f^{-1}(G) \cap f^{-1}(H) \Rightarrow C!$

$\therefore f(X)$ is connected.

Components

A component E of a top. space X is a maximal connected subset of X; that is E is connected and E is not proper subset of any connected subset of X. Clearly E is nonempty

Example 8.12: If X is connected, then X has only one component; X itself.

Example 8.13: Consider the following topology on $X=\{a,b,c,d,e\}$

$T=\{X,\emptyset,\{a\},\{c,d\},\{a,c,d\},\{b,c,d,e\}\}$

The components of X are $\{a\}$ and $\{b,c,d,e\}$. Any other connected subset of X such as $\{b,d,e\}$ is a subset of one the components.

cally connected spaces:

A top. space X is locally connected at $p \in X$ iff every open set containing p contains a connected open set containing p. X is locally connected if it is locally connected at each of its points.

Example 8.14: Every discrete space X is locally connected. For if $p \in X$ then $\{p\}$ is open connected set containing p which is contained in every open set containing p. Note that X is not connected if X contains more th one point.