Let (X,T) is T_1 space then we want < T.p (X,T) is T_0 space>

let $x,y \in X$ then there exist two open sets G and H and $x \neq y$

s.t $x \in G$ and $y \in H$

$y \notin G$ and $x \notin H$

$\because$ $x \in G$ and $y \notin G$ and this mean (X,T) is T_1 space

Remark: The converse is not true as

Example (7.1)

Theorem 7.7: Every T_2 space is T_1 space

proof: Its clear

Remark: The converse is not true

Example 7.9: Let Z the set of integer numbers and T is define as following $T = \{A \subseteq Z : Z - A \text{ is finite}\} \cup \{\phi\}$

Let $a,b \in Z$ then $a < b$ or $a > b$

Let $a < b$ then $\{\dots, a-1, a\} \cup \{b+1, \dots\}$ is open contain a and dont contains b and so $\{\dots, a-1\} \cup \{b, b+1, \dots\}$ is open contain b and dont contains a.

$\Rightarrow (Z,T)$ is T_1 space. every two open in Z has infinite intersection $\Rightarrow (Z,T)$ is not T_2 space.

Theorem 7.8: Every T_3 space is T_2 space

proof: Let (X,T) is T_3 space < T.p (X,T) is T_2 space>

Let $x,y \in X$ s.t $x \neq y$

$\Rightarrow \exists$ open set G s.t $x \in G$ and $y \notin G$ (or converse) since X is T_1 sp

$\Rightarrow F = X \setminus G$ is closed and $y \in F$

and by regularty $\exists$ D is open s.t $y \in F \subseteq D$ and

$x \in H$ s.t $H \cap D = \phi$ $\Rightarrow (X,T)$ is T_2-space

Remark: The converse is not true as the following example

Example 7.10: Let $(\mathbb{R}, T_u)$ is usual top and $D = \{1/n : n = 1, 2$

And

... $S = \{B : B = A - B\}$ when $A \in T_u$ and

$S \subseteq D$ then $(\mathbb{R}, S)$ is top. space

T.p (R, S) is T_2 space

Let $a, b \in \mathbb{R}$ then $a < b$ or $a > b$ when $a \neq b$

let $a < b \Rightarrow \exists c$ s.t $a < c < b$ then

$b \in (c, \infty)$ and $a \in (-\infty, c)$ and $(c, \infty) \cap (-\infty, c) = \phi$

$\Rightarrow (\mathbb{R}, S)$ is T_2 space

Now, T.p $(\mathbb{R}, S)$ is not T_3

Let $H = \cancel{A} - D \Rightarrow H$ is open in S

$\Rightarrow H^c = D$ is closed in S and $0 \notin D$

< T.p there exist no $B_1, B_2 \in T$ s.t $D \subseteq B_1$ and

$0 \in B_2$ and $B_1 \cap B_2 = \phi$ >

Let $B_1, B_2 \in S$ s.t $D \subseteq B_1$

$\Rightarrow B_1 \in T_u \Rightarrow$ every open contain $0 \Rightarrow$ contains

~~element~~ open interval of kind $\frac{1}{n} \Rightarrow B_1 \cap B_2 \neq \phi$

$\Rightarrow (R, S)$ is not T_3-space.

Theorem 7.9: Every T_4 space is T_3 space

proof:

Let (X, T) is T_4-space < T.p (X, T) is T_3 space >

since (X, T) is $T_4 \Rightarrow (X, T)$ is T_1 space

< T.p (X, T) is Regular space >

Let $x \in X$ and $F \subseteq X$ is closed s.t $x \notin F$

since (X, T) is T_1 space $\Rightarrow$ by Theorem 7.1 $\Rightarrow$

$\{x\}$ is closed set and since $\{x\} \notin F$

$\Rightarrow \{x\} \cap F = \phi$ and since (X, T) is T_4-space

$\Rightarrow \exists G$ and H are open sets in T s.t

$\{x\} \subseteq G$ and $F \subseteq H$ and $G \cap H = \phi$

$\Rightarrow x \in G$ and $F \subseteq H$ s.t $G \cap H = \phi$

[illegible] converse above is not true [illegible] since the product of two normal space may be if not normal but the product of two regular space is regular.