$- + \; x \in G$

~~$G^* = \{G_n \in G^*_n : n \in N\}$~~ set of all G^* is form base
~~a base for X since $\mathcal{B}$ is base. Furthermore, $G^* \subset G$ exist~~
~~G^* is countable~~. then there exist countable cover for X.

Separable spaces:

A topological space X is said to be separable if it satisfies the following axiom.

[S] X contains a countable dense subset.

Example 6.8. The real line R with the usual topology is a separable space since the set Q of rational numbers is countable and dence in R.

Example 6.9: $(\mathbb{R}, T_d)$ the dense just R but R is not countable
$\Rightarrow (\mathbb{R}, T_d)$ is not satisfy [S].

Theorem 6.4: If X satisfies the second axiom of countablity, then X is separable.

proof:
Since X satisfies $[C_2]$, X has a countable base $\mathcal{B} = \{B_n : n \in N\}$
$\forall n \in N$ choose $a_n \in B_n$. Then the set $A = \{a_n : n \in N\}$ is countable
$\langle$ T.p $\overline{A} = X \rangle$ or $\langle$ T.p $\forall p \in A^c$ then p is an accumlation point of $A \rangle$.
Let G be an open set containing p. Then G contains at least $B_m \in \mathcal{B}$
Hence $p \in B_m \subset G$ and so $\exists a_m \in A$ s.t $a_m \in B \subset G$
and a_m is different of p since $a_m \in A$ and $p \in A^c$
$\Rightarrow$ p is an accumulation point of A since every open set G containing p also contains a point of A different from p.

Example 6.10: Let Υ be the cofinite top. on any set X. show that (X, T) is separable

Sol:
If X is itself countable then X is separable. If no then we suppose X is not countable then X contains non finite countable subset A but $\overline{A}$ is finite $\Rightarrow \overline{A} = X$

H.w: Show that a discrete space X is separable iff X is countable.

Definition 6.1: A property p of a top. space X is said to be (heredilitary) if every ~~subset~~ subspace of X (relative top.) possesses prop p.

$[C_1]$ is hereditary (exp 6.3)
(exp 6.7).

E... ,tion axioms: — chapter ~~seven~~ ...

... pological space X is a T_0-space iff satisfies the following ...iom:

[T_0] $\forall x, y \in X$, $x \neq y$, $\exists G$ open s.t $x \in G$ and $y \notin G$ or $x \notin G$ and $y \in G$.

Example (7.1): Let $X = \{a, b\}$ and $T = \{\phi, X, \{a\}\}$ then $a \neq b$ but $\exists G$ open $\{a\}$ s.t $a \in \{a\}$ and $b \notin \{b\}$.

[T_1] ~~for any~~ $\forall x, y \in X$, $x \neq y$, $\exists G$ and H are open sets s.t $x \in G$ and $y \notin G$, $x \notin H$ and $y \in H$

Example 7.2: Let $X = \{a, b\}$ and $T = \{\phi, X, \{a\}, \{b\}\}$ then $a \neq b$

$a \in \{a\}$ and $b \notin \{a\}$

$a \notin \{b\}$ and $b \in \{b\}$

Theorem 7.1: A topological space X is a T_1-space iff every singleton subset $\{p\}$ of X is closed.

proof: $\langle$ T.P $\{p\}^c$ is open $\rangle$

Let X is T_1-space and $p \in X$ and since (X, T) is $T_1 \Rightarrow \exists G_x$ open

Let $x \in \{p\}^c$ then $x \neq p$ $\Rightarrow G_x \subset \{p\}^c \Rightarrow \{p\}^c$ is open $\Rightarrow$

and $x \in G_x$ $p \notin G_x$

$\{p\}$ is closed. $\forall p \in X$. Let $a, b \in X$ s.t $a \neq b$

Let $\{p\}$ is closed and so $a \notin \{a\}^c$ and $b \notin \{b\}^c$

$\Rightarrow a \in \{b\}^c$ and $b \in \{a\}^c$

$\Rightarrow (X, T)$ is T_1-space.

Example 7.3: In indescrete (X, T), every set is open and closed therefore $\{p\}$, $p \in X$ is closed $\Rightarrow (X, T)$ is T_1-space.

[T_2] or Hausdorff space.

$\forall x, y \in X$, $x \neq y$, $\exists G, H$ open sets s.t $x \in G$, $y \in H$ and $G \cap H = \phi$

Example 7.4: indescrete top. space (X, T) is T_2-space or Hausdorff space since $\forall a, b \in X$ $a \neq b$

$a \in \{a\}$ and $b \in \{b\}$ and $\{a\} \cap \{b\} = \phi$