((المحاضرة الأولى))

Introduction :-

We need the study of complex numbers because of the results of some algebraic opeations on the real numbers, are not real numbers. Such that some algebraic equations dont have solutions in the set of real numbers.

the study of cmplex numbers is extension of the studying of real numbers. In this study, we can ... the algebraic construction of the complex numbers and the geometric representation of it in the plan. further more study of the fundamatal algebraic operations o... it the system of complex numbers started on the

Solution of general equation of second degree:

$$ax^2+bx+c=0$$

The Solution of this equation is

$$x=\frac{-b\mp\sqrt{b^2-4ac}}{2a}$$

When $b^2-4ac\geq 0$, then the roots of the equations are reals but when $b^2-4ac<0$, then there is not mean of the square root and we say in this case, there is not solution of the equation in the system of real numbers.

For example:-

There are not real solution of the equation.

$$x^2+2x+3=0.$$

There fore determined another system of numbers, hes Solution of the equation such that they are not met.

1) Contain all of the real numbers, hence system of numbers bigger of the real numbers.

2) Contain the number i (it is not real numbers) with the property $i^2=-1$.

We say for this system is the system of complex numbers. Such that play in it the number $i=\sqrt{-1}$ basic this system. We can representiation the complex numbers as point in plan.

Note:

1) $(i\sqrt{9})^2 = i^2 \cdot (\sqrt{9})^2 = -1 \cdot 9 = -9$.

2) $(-i\sqrt{9})^2 = (-i)^2 \cdot (\sqrt{9})^2 = -1 \cdot 9 = -9$.

Complex numbers

Definition:- A complex numbers z can be defined as ordered pair of real number x, y that satisfies certian laws of operation.

It is written in either of the two forms:

$$z = x + iy \quad \text{or} \quad z = x + yi$$

where $i = \sqrt{-1}$. when $y = 0$ the complex number z become the real number x, that is, complex numbers all the real numbers, $x + i0 = x$.

The real multiples of i, $yi = 0 + iy = yi$ are called pure imaginary numbers.

when $y \neq 0$, the complex number $x + iy$ is often called an imaginary number. the real numbers x and y of are known as the real and imaginary components of the complex number z.

They are also called the real part and imaginary part of z. We denote the real part and imaginary part as follows:

$$Re(z) = x \quad , \quad Im(z) = y .$$

Note: Two complex numbers $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$ are equal if and only if their real parts are equal and their imaginary parts are equal

$$z_1 = z_2 \text{ implies } x_1 = x_2 \, , \, y_1 = y_2 .$$

Fundamental operations العمليات الأساسية

Two further operations, the laws of addition and multiplication are be included in the definition of Complex numbers.

Let z_1 and z_2 be any two complex numbers

$$z_1 = x_1 + iy_1 \quad , z_2 = x_2 + iy_2$$

then $z_1 + z_2 = (x_1 + iy_1) + (x_2 + iy_2)$

$$= (x_1 + x_2) + i(y_1 + y_2) .$$

the product of z_1 and z_2 given by the form:

$$z_1 z_2 = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1) .$$

We can write $-z = (-1z)$ such that

$$-z = -x - iy .$$

Hence $-z$ is a complex number such that

$$z + (-z) = 0 .$$

There fore, we can define the operation of subtraction between the complex numbers as following:

$z_1 - z_2 = z_1 + (-z_2) = (x_1 - x_2) + i(y_1 - y_2)$.

the real number 0 and 1 an identity elements for addition and multiplication operation respectively in $\mathbb{C}$.

Hence $0 + z = z$ and $1 \cdot z = z \; \forall \; z \in \mathbb{C}$.

We can defined the division two complex numbers z_1, z_2 writen as follows: $\frac{z_1}{z_2}$ if $z_2 \neq 0$ and $\frac{z_1}{z_2} = w$, then $z_1 = w z_2$. Such that $w = u + iv$.

Hence $x_1 + iy_1 = (u + iv)(x_2 + iy_2)$
$= (ux_2 - vy_2) + i(x_2 v + uy_2)$.

by solving the equations.

$$ux_2 - vy_2 = x_1$$
$$uy_2 + vx_2 = y_1$$

: we get $\frac{z_1}{z_2} = \frac{x_1x_2 + y_1y_2}{x_2^2+y_2^2} + i\,\frac{y_1x_2 - x_2y_2}{x_2^2+y_2^2}$

when $z_1=1$, $z_2=z$, we get

$$\frac{1}{z} = \frac{1}{x+iy} = \frac{x}{x^2+y^2} + i\frac{-y}{x^2+y^2}.$$

Some properties of the Complex Numbers

1) $z_1+z_2 = z_2+z_1$, $z_1 \cdot z_2 = z_2 \cdot z_1$ ((Commutative)) H.W

2) $(z_1+z_2)+z_3 = z_1+(z_2+z_3)$, $(z_1z_2)z_3 = z_1(z_2z_3)$ (Associative) H.W

3) $z_1(z_2+z_3) = z_1z_2 + z_1z_3$ (Distributive) H.W.

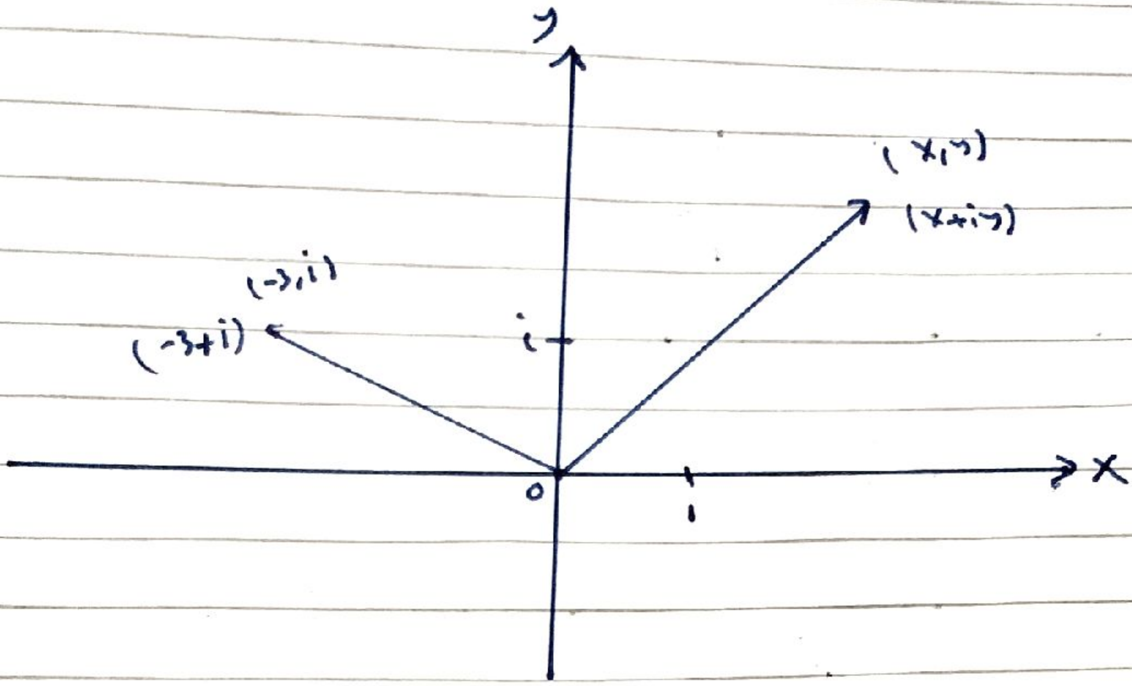
Note:

the invers of the complex numbers z of additive

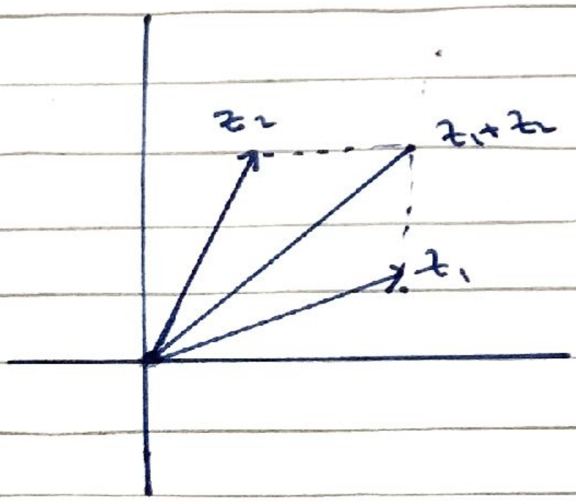
and multiplication aperation are $(-z)$ and $(\frac{1}{z} = z^{-1})$

respectively such that $z+(-z)=0$ and $z z^{-1} = 1$

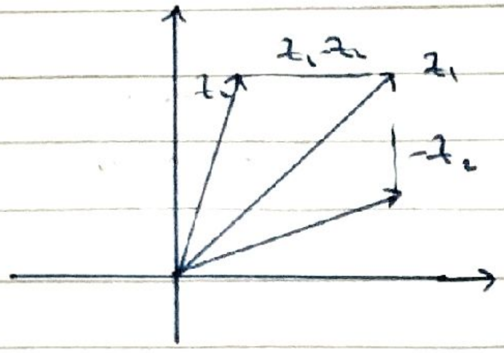
Numbers by point on the plan.



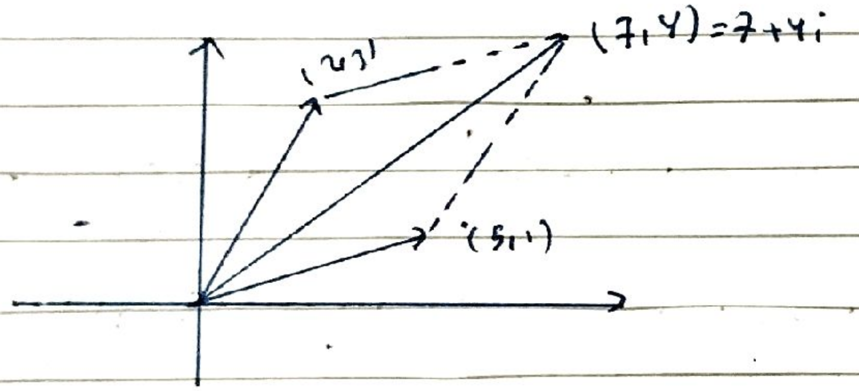
Hence z_1+z_2 is represented by vector sum of the vectors z_1 and z_2. as shown in next figuare.



The difference $z_1 - z_2$ is represented by vector



ex: – $z_1 = 2+3i$, $z_2 = 5+i$, then



$z_1 + z_2 = (2+3i) + (5+i) = 7+4i$

H.W
ex: If $z_1 = 3+6i$, $z_2 = 2+i3$, find $z_1 - z_2$ and

Show the figuare.

Exercises:

1) Calculate $(1+2i)^3$

2) find the geometric representation of the following numbers $z_1 z_2$, $z_1 + z_2$, $z_1 - z_2$ where $z_1 = 1+2i$, $z_2 = 2-i$