ممكن يشمل الامتحان

② find $\lim\limits_{z\to\infty} \dfrac{2z+i}{z+1}$?

Solu//

$$\lim_{z\to 0} \frac{\frac{2}{z}+i}{\frac{1}{z}+1} = \lim_{z\to 0} \frac{2+iz}{1+z} = 2.$$

$$\therefore \lim_{z\to\infty} \frac{2z+i}{z+1} = 2.$$

③ find $\lim\limits_{z\to\infty} \dfrac{2z^3-1}{z^2+1}$?

Solu//

$$\lim_{z\to 0} \frac{1}{\left(\dfrac{2(\frac{1}{z})^3-1}{(\frac{1}{z})^2+1}\right)}$$

$$= 0.$$

then $\lim\limits_{z\to\infty} \dfrac{2z^3-1}{z^2+1} = \infty.$

theorem :- If $\lim\limits_{z\to z_0} f(z)$ is exists, then the Limit is unique ?

theorem:- If $f(z) = u(x,y) + iv(x,y)$, then

$\lim_{z \to z_0} f(z) = w_0 = u_0 + iv_0$ if and only if

$\lim_{\substack{x \to x_0 \\ y \to y_0}} v(x,y) = v_0$, $\lim_{\substack{x \to x_0 \\ y \to y_0}} u(x,y) = u_0$.

theorem:-

① If $f(z), g(z)$ are functions and

$\lim_{z \to z_0} f(z) = L$, $\lim_{z \to z_0} g(z) = M$, then

ⓐ $\lim_{z \to z_0} [f(z) \pm g(z)] = L \pm M$.

ⓑ $\lim_{z \to z_0} [f(z) \cdot g(z)] = L \cdot M$.

ⓒ $\lim_{z \to z_0} \frac{f(z)}{g(z)} = \frac{L}{M}$, $M \neq 0, g(z) \neq 0$

ⓓ $\lim_{z \to z_0} |f(z)| = |L|$.

② $\lim_{z \to z_0} C = C$, s.t. C is constant.

③ $\lim_{z \to z_0} z^n = z_0^n$, $n \in \mathbb{Z}^+$.

proof // H.W.

الاتصال

* Continuity الاتصالية

Def:- Let $f(z)$ be a function defined through neighborhood of point z_0, we say that f is continuus function in z_0 if $\forall \epsilon > 0$ $\exists$ $\overset{\text{جوار}}{N}$ to z_0 with radius $\delta > 0$ s.t.

$$|f(z) - f(z_0)| < \epsilon, \text{ when } 0 < |z - z_0| < \delta.$$

In other words: The function f is continuous in z_0 if all three of the following conditions are satisfied:

1. $f(z_0)$ exists
2. $\lim_{z \to z_0} f(z)$ exists
3. $\lim_{z \to z_0} f(z) = f(z_0)$.

Def:- A function f is cont's in the region R if and only if the function f is cont's of each point of the points of region R.

ex:- A function $f(z)=z^2$ is always cont's in the complex plan, since $f(z_0)=z_0^2$ and

$\lim_{z \to z_0} f(z) = f(z_0)$.

ex:- A function $f(z)=\frac{z^2-1}{z-1}$ is not cont's at $z=1$, because $f(1)$ is not defined.

ex:- prove that $f(z)=2z+3$ is cont's for all z_0

sol// let $z_0 \in \mathbb{C}$

$\forall \epsilon>0, \exists \delta>0$ s.t. $|f(z)-f(z_0)|<\epsilon$, whene $|z-z_0$

then $|f(z)-f(z_0)| = |2z+3-(2z_0+3)|$

$= |2z+3-2z_0-3|$

$= |2(z-z_0)| = 2|z-z_0| < \epsilon$

$\therefore |f(z)-f(z_0)| < \epsilon$

$\Rightarrow 2|z-z_0| < \epsilon \Rightarrow |z-z_0| < \frac{\epsilon}{2}$

So $\delta = \frac{\epsilon}{2}$.

Theorem :- If $f(z) = u(x,y) + i v(x,y)$ is a Cont's at $z_0 = x_0 + i y_0$ if and only if $u(x,y)$ and $v(x,y)$ are Cont's functions at (x_0, y_0).

Theorem :- Let $f(z)$ is a Cont's at z_0, then $|f(z)|$ is Cont's at z_0.

Theorem :- A function of polynomial from degree n, such that

$$f(z) = a_0 + a_1 z + \dots + a_n z^n, \quad a_n \neq 0$$ is

a Cont's at each point of points of the complex pl[ane]

Theorem :- If $f(z)$, $g(z)$ are Cont's functions at z_0, then

ⓐ $f(z) \mp g(z)$

ⓑ $f(z) . g(z)$

ⓒ $\dfrac{f(z)}{g(z)}$, $g(z) \neq 0$

are Cont's functions.

Theorem:- A compisition of cont's functions is a cont's.