* Functions of Complex variable

Let S be a set of coplex numbers. A function f defind on S is assgas to each z in a Complex number w. The number w is called the value of f at z and is denoted by $f(z)$. s.t $w = f(z)$. The set S is called the domin of definition of f, and f is called Complex function. like

1) $w = f(z) = z^2 + 2z$

2) $w = f(z) = |z+3|$

Def: ((Single ~~function~~ value)) function

The function $w = f(z)$ is called single ~~function~~ value function in S if for each $z \in \mathbb{C}$, there one value from $w = f(z)$.

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ex//

$w = f(z) = z^2 + 2z$

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Def: (multi-valued function)

the function $w = f(z)$ is called (m.v.f) in S

if for each $z \in \mathbb{C}$, there multi-value of $w = f(z)$.
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ex; $w = f(z) = z^{\frac{1}{3}}$ is triple values since, for
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each z, there three value of w.

$w = f(z) = z^{\frac{1}{3}}$, $w = r^{\frac{1}{3}} e^{i\left(\frac{\theta + 2k\pi}{3}\right)}$, $k = 0, 1, 2, \ldots$

s.t.

$z = re^{i\theta}$,

Note, Let u, v are real part and Im. part
(Re) (Im)

of the function $w = f(z)$ resp.

s.t

$w = f(z) = u(x,y) + iv(x,y)$

then $w = u + iv$.

like, we can be rewrite the function $w = f(z) = z^2$

by form:

$w = f(z) = f(x+iy) = (x+iy)^2$

$w = f(z) = x^2 - y^2 + 2ixy$

...h that $u(x) = x^2 - y^2$, $v = 2xy$

Note//

① $p(z) = a_0 + a_1 z + \cdots + a_n z^n$ with $a_n \neq 0$ is a polynomial of degree n.

② Quotient $\frac{p(z)}{q(z)}$ of polynomials $p(z)$ and $q(z)$ is called rational function (defined at z with $q(z) \neq 0$)

• using polar coordinates a exp. fun. of z:

$$u = u(r, \theta)$$
$$v = v(r, \theta)$$

So, we write $f(z) = u(r,\theta) + i v(r,\theta)$, $z = re^{i\theta}$

ex// $w = f(z) = z^2$ with $z = re^{i\theta}$

$$w = (re^{i\theta})^2 = r^2 e^{i2\theta} = r^2(\cos 2\theta + i\sin 2\theta)$$
$$u = r^2 \cos 2\theta$$
$$v = r^2 \sin 2\theta$$