Linear tranformation:

Def: - If $T: V \longrightarrow W$ is a function form avector space V to avector space W, then T is called a linear tranformation form V to W if the following two Properties:-

1- $T(v+u) = T(u) + T(v)$

2- $T(Ku) = KT(u)$ where is real number

Ex:- if it was $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ Know difined function is T a linear tranformation

$T(x, y) = (y, x+2y, 2x-y)$

Sol:- let $u = (x_1, y_1)$, $v = (x_2, y_2)$

$u + v = (x_1, y_1) + (x_2, y_2)$

$= (\underbrace{x_1 + x_2}_{x}, \underbrace{y_1 + y_2}_{y})$

$T(u, y) = (y_1 + y_2, x_1 + x_2) + 2(y_1 + y_2), 2(x_1 + x_2) - (y_1 + y_2)$

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$= y_1 + y_2 , x_1 + x_2 + 2y_1 + 2y_2 , 2x_1 + 2x_2 - y_1 - y_2$

$T(u) = y_1 , x_1 + 2y_1 , 2x_1 - y_1$

$T(v) = y_2 , x_2 + 2y_2 , 2x_2 - y_2$

$T(u) + T(v) = (y_1 , x_1 + 2y_1 , 2x_1 - y_1) + (y_2 , x_2 + 2y_2 , 2x_2 - y_2)$

$Tu + Tv = (y_1 + y_2 , (x_1 + x_2) + 2y_1 + 2y_2 , 2x_1 + 2x_2 - y_1 - y_2)$

$\therefore T(u+v) = T(u) + T(v)$

2- $T(Ku) = T(K(x_1 , y_1)) = T(Kx_1 , Ky_1)$

$= (Ky_1 , Kx_1 + 2Ky_1 , 2Kx_1 - Ky_1)$

$= (Ky_1 , K(x_1 + 2y_1) , K(2x_1 - y_1))$

$= K(y_1 , x_1 + 2y_1 , 2x_1 - y_1)$

$= K\,T(u)$

$\therefore T(Ku) = K\,T(u)$ is linear $\therefore$ Tranformation.

Ex: Is $T(x,y) = (x, y+1)$ be a linear transform? why?

Sol:

let: $u = (x_1, y_1)$, $v = (x_2, y_2)$

$u + v = (x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$

$T(u+v) = (x_1 + x_2, \overset{y_1+2}{y_1} + 1)$

$T(u) = (x_1, y_1 + 1)$

$T(v) = (x_2, y_2 + 1)$

$T(u) + T(v) = (x_1 + x_2, y_1 + y_2 + 2)$

$T\ (x_1, y_1) + (x_2, y_2)$

$T(u+v) \neq T(v) + T(u)$ is not linear transformation.

Remark: If A is $m \times n$ matrix over $\mathbb{R}$ then we can

difine $T: \mathbb{R}^n \longrightarrow \mathbb{R}^m$

s.t $T(x) = A_{m\times n} \, X_{n\times 1}$, $x \in \mathbb{R}^n$

matrix of $n\times 1$ over F and AX matrix of $m\times 1$ over

$\mathbb{R}$ is called matrix transform.

Ex: If $A = \begin{bmatrix} 4 & 5 \\ 6 & 2 \end{bmatrix}_{2\times 2}$ find matrix tran form. or $T(x) = AX$

Sol: let $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

$T(A) = AX_{2x}$

$$T(x,y) = \begin{bmatrix} 4 & 5 \\ 6 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$T(x,y) = \begin{bmatrix} 4x + 5y \\ 6x + 2y \end{bmatrix}$$

Ex: If $A = \begin{bmatrix} 3 & 2 & 4 \\ 5 & 2 & 6 \end{bmatrix}_{2\times3}$ find $T(x) = AX$

Sol: $T: R^3 \longrightarrow R^2$

$T(A) = A_{2\times3} X_{3\times1} \Rightarrow T(x) = \begin{pmatrix} 3 & 2 & 4 \\ 5 & 2 & 6 \end{pmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$= \begin{bmatrix} 3x + 2y + 4z \\ 5x + 2y + 6z \end{bmatrix}$

Ex: let $T: R^3 \longrightarrow R^2$ find Matrix tranform satisfies the following

$A_{2\times3}$

$T\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$T: R^3 \longrightarrow R^2$

$T\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$T\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

find $T\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ the find matrix transform.?

$\Longleftarrow$

let: $T: M_{2\times2}(\mathbb{R}) \longrightarrow M_{2\times2}(\mathbb{R})$

$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} 0 & b \\ 0 & d \end{bmatrix}$. Is be linear Tran.?

Sol: $A = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$, $B = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$

① $T(A+B) = T\left(\begin{bmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{bmatrix}\right) = \begin{pmatrix} 0 & b_1+b_2 \\ 0 & d_1+d_2 \end{pmatrix}$

$T(A)+T(B) = T\left(\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}\right) + T\left(\begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}\right)$

$= \begin{bmatrix} 0 & b_1 \\ 0 & d_1 \end{bmatrix} + \begin{bmatrix} 0 & b_2 \\ 0 & d_2 \end{bmatrix} = \begin{bmatrix} 0 & b_1+b_2 \\ 0 & d_1+d_2 \end{bmatrix}$

$\therefore T(A+B) = T(A) + T(B)$

2) $T(KA) = K\,T(A)$

$T\left(\begin{bmatrix} Ka_1 & Kb_1 \\ Kc_1 & Kd_1 \end{bmatrix}\right) = \begin{bmatrix} 0 & Kb_1 \\ 0 & Kd_1 \end{bmatrix}$

$K\,T(A) = K\,T\left(\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}\right)$

$$= K \begin{bmatrix} 0 & b_1 \\ 0 & d_1 \end{bmatrix} = \begin{bmatrix} 0 & Kb_1 \\ 0 & Kd_1 \end{bmatrix}$$

$\therefore T(KA) = K\,T(A)$

Hence T is leaner Tran.

the Kernel and the range

The Kernel / If $T: V \to W$ is L.T.

then $Ker(T) = \{\forall x \in V : T(x) = 0\}$

the Image / If $T: V \to W$ is L.T

$$Im(T) = \{B \in V : T(B)\}$$

let $L: R^3 \longrightarrow R^2$ be the linear Tran

such that $L(x, y, z) = (x, y)$ find Ker L?

Sol:-

$$\text{Ker } L = \{(x, y, z) \in R^3 : L(x, y, z) = 0_{R^2}\}$$

$$= \{(x, y, z) \in R^3 : (x, y) = (0, 0)\}$$

$$= \{(0, 0, z) \in R^3 : z \in R\}$$

theorem:- If $T: V \longrightarrow W$ is L.T then

1- Ker(T) is subSpace of V 2- Im(T) is subSpace of W

Proof:- or $Im(T) = \{ B \in W : T(A) = B ; \forall \in V \}$

$T(0) = 0$

then $0 \in Ker(T) \Rightarrow Ker(T) \neq 0$

let $x, y \in Ker(T) \xrightarrow{T.P} x + y \in Ker(T)$

$\therefore T(x+y) = 0 \Longrightarrow x + y\ Ker(T)$

2- let $x \in Ker\ T$, and $K \in f$

$T(KX) = 0$

$KX \in Ker(T)$

$\therefore$ Ker T is sub space of V.

Ex: let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ is linear Tran.

$T(x,y,z) = (x+y,0,2z)$ find $Ker(T)$, $Im(T)$

Sol:- $Kel(T) = \{x \in \mathbb{R}^3 ; T(x) = 0\}$

$= \{(x,y,z) \in \mathbb{R}^3 ; T(x,y,z) = (0,0,0)\}$

$= \{(x,y,z) \in \mathbb{R}^3 : (x+y,0,2z) = (0,0,0)\}$

$= \{(-y,y,0) \in \mathbb{R}^3\} = \mathbb{R}^3$

$Im(T) = \{(x,y,z) \in \mathbb{R}^3 \mid T(x,y,z) = \forall (x,y,z) \in \mathbb{R}^3$

$= \{(x,y,z) \in \mathbb{R}^3 \mid (x+y,0,2z) ; \forall (x,y,z) \in \mathbb{R}^3$