1. GRAPHICAL CONSTRUCTIONS

THE PARALLEL-RAY METHOD

It would be well to point out here that although the above formulas hold for all possible object and image distances, they apply only to images formed by paraxial rays.

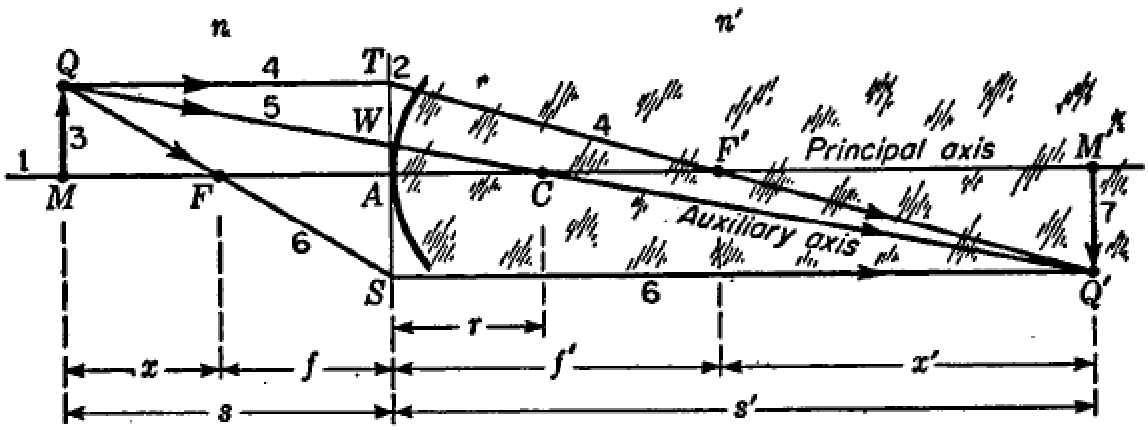

Fig. 1: Parallel-ray method for graphically locating the image formed by a single spherical surface.

The parallel-ray method of construction is illustrated in Figs. 1 and 2 for convex and concave surfaces, respectively. Consider the light emitted from the highest point Q of the object in Fig. 1. Of the rays emanating from this point in different directions the one (QT) traveling parallel to the axis will by definition of the focal point be refracted to pass through $F^{\backslash}$. The ray QC passing through the center of curvature is undeviated because it crosses the boundary perpendicular to the surface. These two rays are sufficient to locate the tip of the image at *Q'*, and the rest of the image lies in the conjugate plane through this point. All other paraxial rays from *Q*, refracted by the surface, will also be brought to a focus *Q'*. The ray *QS*, which passes through the point *F*, will be refracted parallel to the axis and will cross the others at *Q'* as shown in the figure. This method is called the *parallel-ray method.* The numbers 1, 2, 3, ... indicate the order in which the lines are drawn.

When the method just described is applied to a diverging system, as shown in Fig. 2, similar procedures are carried out. Ray *QT*, drawn parallel to the axis, is refracted as if it came from *P'*. Ray *Q*S, directed toward *F*, is refracted parallel to the axis. Finally ray *Q W*, passing through C, goes on undeviated. Extending all these refracted rays back to the left finds them intersecting at the common point *Q'*. *Q'M'* is therefore the image of the object *QM*. Note that *Q'M'* is not a real image since it cannot be formed on a screen.

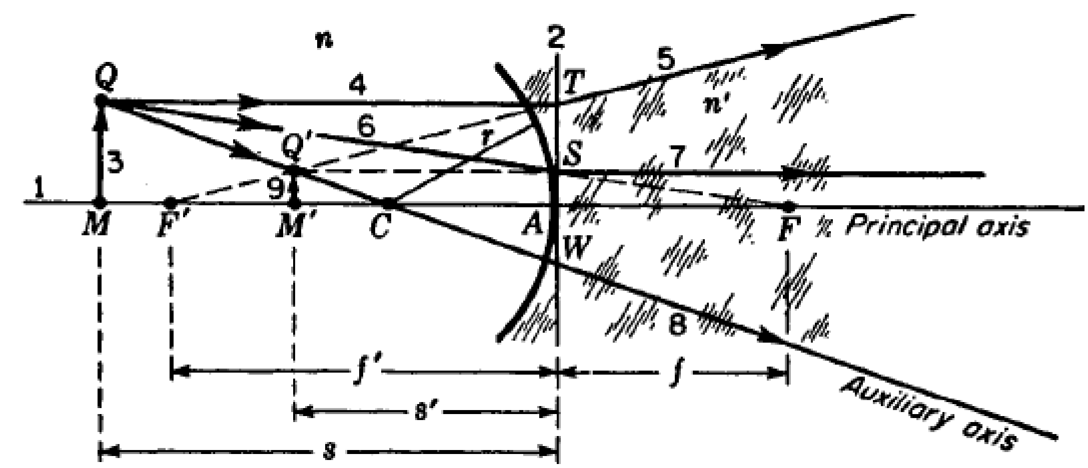

Fig.2: The parallel-ray method applied to a concave spherical surface having diverging properties.

In both these figures the medium to the right of the spherical surface has the greater index; i.e., we have made $n' > n$. Since any ray through the center of curvature is undeviated and has all the properties of the principal axis, it may be called an *auxiliary axis.*

2. OBLIQUE-RAY METHODS

Method 1 In these constructions one is free to choose any two rays coming from a common object point and, after tracing them through the system, find where they finally intersect. This intersection is then the image point. Let MT in Fig. 3 represent any ray incident on the surface from the left. Through the center of curvature C a dashed line RC is drawn, parallel to MT, and extended to the point where it crosses the secondary focal plane. The line TX is then drawn as the refracted ray and extended to the point where it crosses the axis at M'. Since the axis may here be considered as a second ray of light, M represents an axial object point and M' its conjugate image point. The principle involved in this construction is the following. If MT and RA were parallel incident rays of light, they would (after refraction and by the definition of focal planes) intersect the secondary focal plane WF' at X. Since RA is directed toward C, the refracted ray ACX remains undeviated from its original direction.

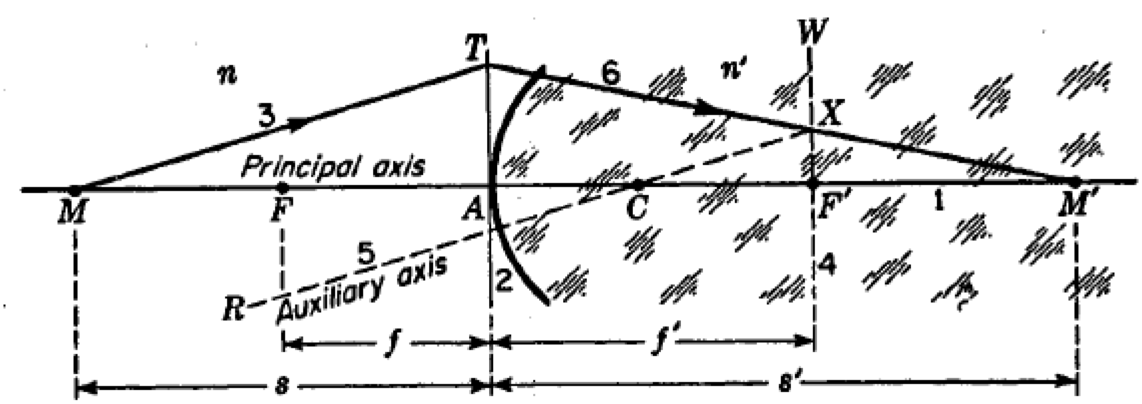

Fig.3: The oblique-ray method for graphically locating images formed by a single spherical surface.

Method 2 This method is shown in Fig. 4. After drawing the axis MM' and the arc representing the spherical surface with a center C, any line such as 1 is drawn to represent any oblique ray of light. Next, an auxiliary diagram is started by drawing XZ parallel to the axis. With an origin at 0, line intervals OK and OL are laid off proportional to n and n', respectively, and perpendiculars are drawn through K, L, and A. From here the construction proceeds in the order of the numbers 1, 2, 3, 4,5, and 6. Line 2 is drawn through 0 parallel to line I, line 4 is drawn through J parallel to line 3, and line 6 is drawn through T parallel to line 5.

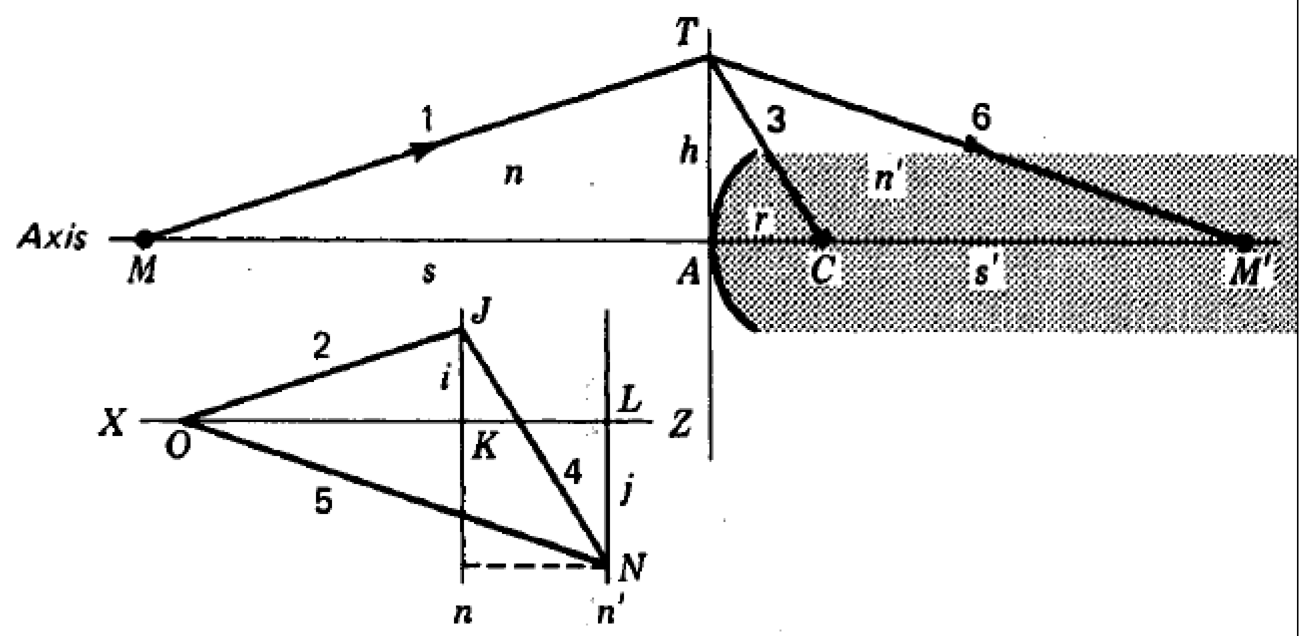

Fig. 4: The auxiliary-diagram method for graphically locating images formed by paraxial rays.

A proof for this construction is readily obtained by writing down proportionalities from three pairs of similar triangles in the two diagrams. These proportionalities are:

$$\frac{h}{s} = \frac{i}{n} \qquad , \frac{h}{s'} = \frac{j}{n'} \qquad , \quad \frac{h}{r} = \frac{i + j}{n' - n} \tag{1}$$

Transpose *n* and *n*' to the left in all three equations,

$$\frac{hn}{s} = i \qquad , \frac{h\,n'}{s'} = j \qquad , \quad \frac{h\,(n' - n)}{r} = i + j \tag{2}$$

Finally, add the first two equations and for the right-hand side substitute the third equality:

$$\frac{hn}{s} + \frac{h\,n'}{s'} = i + j \qquad and \qquad \frac{n}{s} + \frac{n'}{s'} = \frac{n' - n}{r} \tag{3}$$

3. MAGNIFICATION

In any optical system the ratio between the transverse dimension of the final image and the corresponding dimension of the original object is called the lateral magnification. To determine the relative size of the image formed by a single spherical surface, reference is made to the geometry of Fig. 1. Here the undeviated ray 5 forms two similar right triangles QMC and Q'M'C. The theorem of the proportionality of corresponding sides requires that

$$\frac{M'\,Q'}{M\,Q} = \frac{C\,M'}{C\,M} \qquad or \qquad \frac{-\,y'}{y} = \frac{s' - r}{s + r} \tag{4}$$

We now define *y' /y* as the lateral magnification *m* and obtain

$$m = \frac{y'}{y} = -\,\frac{s' - r}{s + r} \tag{5}$$

If *m* is positive, the image will be virtual and erect, while if it is negative, the image is real and inverted.

4. DERIVATION OF THE GAUSSIAN FORMULA

The basic equation (*) is importance to derivative in some detail. A method involving oblique rays will be given here. In Fig. 5 an oblique ray from an axial object point M is shown incident on the surface at an angle ϕ and refracted at an angle ϕ'. The refracted ray crosses the axis at the image point M'. If the incident and refracted rays MT and T M' are paraxial, the angles ϕ and ϕ' will be small enough to permit putting the sines of the two angles equal to the angles themselves; for Snell's law we write

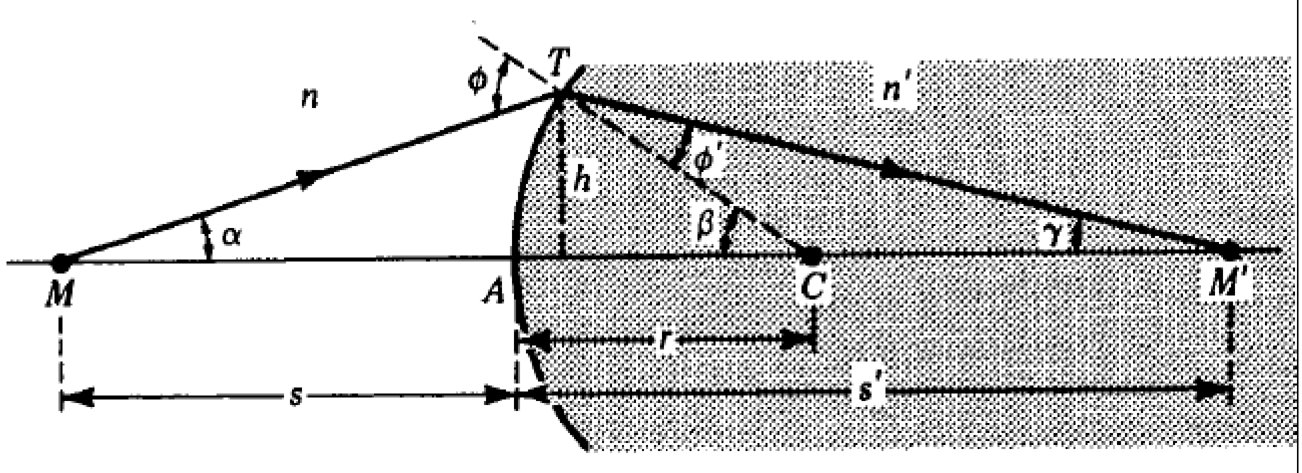

Fig. 5: Geometry for the derivation of the paraxial formula used in locating images.

- $$\frac{n}{s} + \frac{n'}{s'} = \frac{n' - n}{r}$$

$\frac{\emptyset}{\emptyset^{\backslash}} = \frac{n^{\backslash}}{n}$	6

Since ϕ is an exterior angle of the triangle *MTC* and equals the sum of the opposite interior angles, $\phi = \alpha + \beta$ -------- (7)

Similarly β is an exterior angle of the triangle *TCM'*, so that $\beta = \phi' + \gamma$ and

$\phi' = \beta - \gamma$ ---------------- (8)

Substituting these values of ϕ and ϕ' in Eq. (6) and multiplying out, we obtain

$n^{\backslash}\beta - n^{\backslash}\gamma = n\,\alpha + n\,\beta \quad or\ n\alpha + n^{\backslash}\,\gamma = (n^{\backslash} - n)\beta$	9

For paraxial rays α, β, and γ are very small angles, and we may set $\alpha = h / s$, $\beta = h / r$, and $\gamma = h / s'$. Substituting these values in the last equation, we obtain

$n\,\frac{h}{s} + n'\,\frac{h}{s'} = (\,n' - n\,)\,\frac{h}{r}$	10

By canceling *h* throughout we obtain the desired equation,

$\frac{n}{s} + \frac{n'}{s'} = \frac{n' - n}{r}$	11