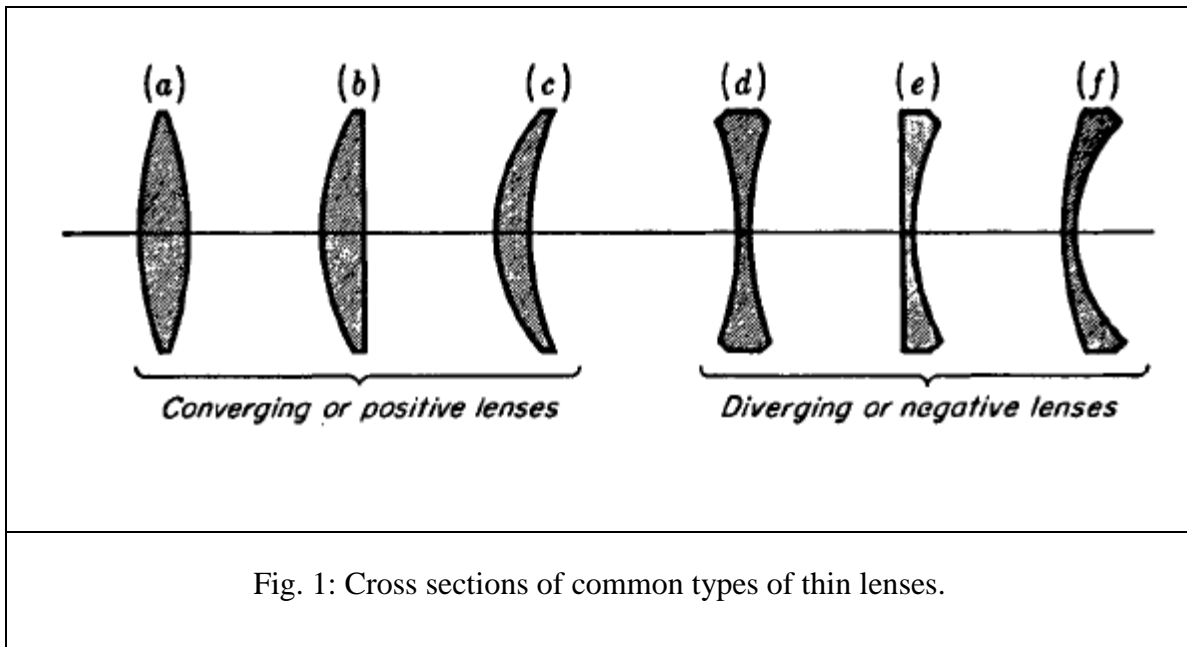


1. SPHERICAL SURFACES

Many common optical devices contain not only mirrors and prisms having flat polished surfaces but lenses having spherical surfaces with a wide range of curvatures. Such spherical surfaces, in contrast with plane surfaces treated in the last chapter, are capable of forming real images. Cross-sectional diagrams of several standard forms of lenses are shown in Fig. 1. The three *converging*, or *positive*, *lenses*, which are thicker at the center than at the edges, are shown as (a) *equiconvex*, (b) *plano-convex*, and (c) *positive meniscus*. The three *diverging*, or *negative*, *lenses*, which are thinner at the center, are (d) *equiconcave*, (e) *plano-concave*, and (f) *negative meniscus*. Such lenses are usually made of optical glass as free as possible from inhomogeneities, but sometimes other transparent materials like quartz; fluorite, rock salt, and plastics are used.



2. FOCAL POINTS AND FOCAL LENGTHS

Characteristic diagrams showing the refraction of light by convex and concave spherical surfaces are given in Fig. 2. The principal axis in each diagram is a straight line through the center of curvature C . The point A where the axis crosses the surface is called the *vertex*. In diagram (a) rays are shown diverging from a point source F on the axis in the first medium and refracted into a beam everywhere parallel to the axis in the second medium. Diagram (b) shows a beam converging in the first medium toward the point F and then refracted into a parallel beam in the second medium. F in each of these two cases is called the *primary focal point*, and the distance f is called the *primary focal length*.

In diagram (c) a parallel incident beam is refracted and brought to a focus at the point F' , and in diagram (d) a parallel incident beam is refracted to diverge as if it came from the point P . F' in each case is called the *secondary focal point*, and the distance f' is called the *secondary focal length*.

Returning to diagrams (a) and (b) for reference, we now state that *the primary focal point F is an axial point having the property that any ray coming from it or proceeding toward it travels parallel to the axis after refraction*. Referring to diagrams (c) and (d), we make the similar statement that *the secondary focal point F' is an axial point having the property that any incident ray traveling parallel to the axis will, after refraction, proceed toward, or appear to come from, F'* .

A plane perpendicular to the axis and passing through either focal point is called a *focal plane*. The significance of a focal plane is illustrated for a convex surface in Fig. 2. Parallel incident rays making an angle θ with the axis are brought to a focus in the focal plane at a point Q' . Note that Q' is in line with the un deviated ray through the center of curvature C and that this is the only ray that crosses the boundary at normal incidence.

It is important to note in Fig. 2 that the primary focal length f for the convex surface [diagram (a)] is not equal to the secondary focal length f' of the same surface [diagram (c)].

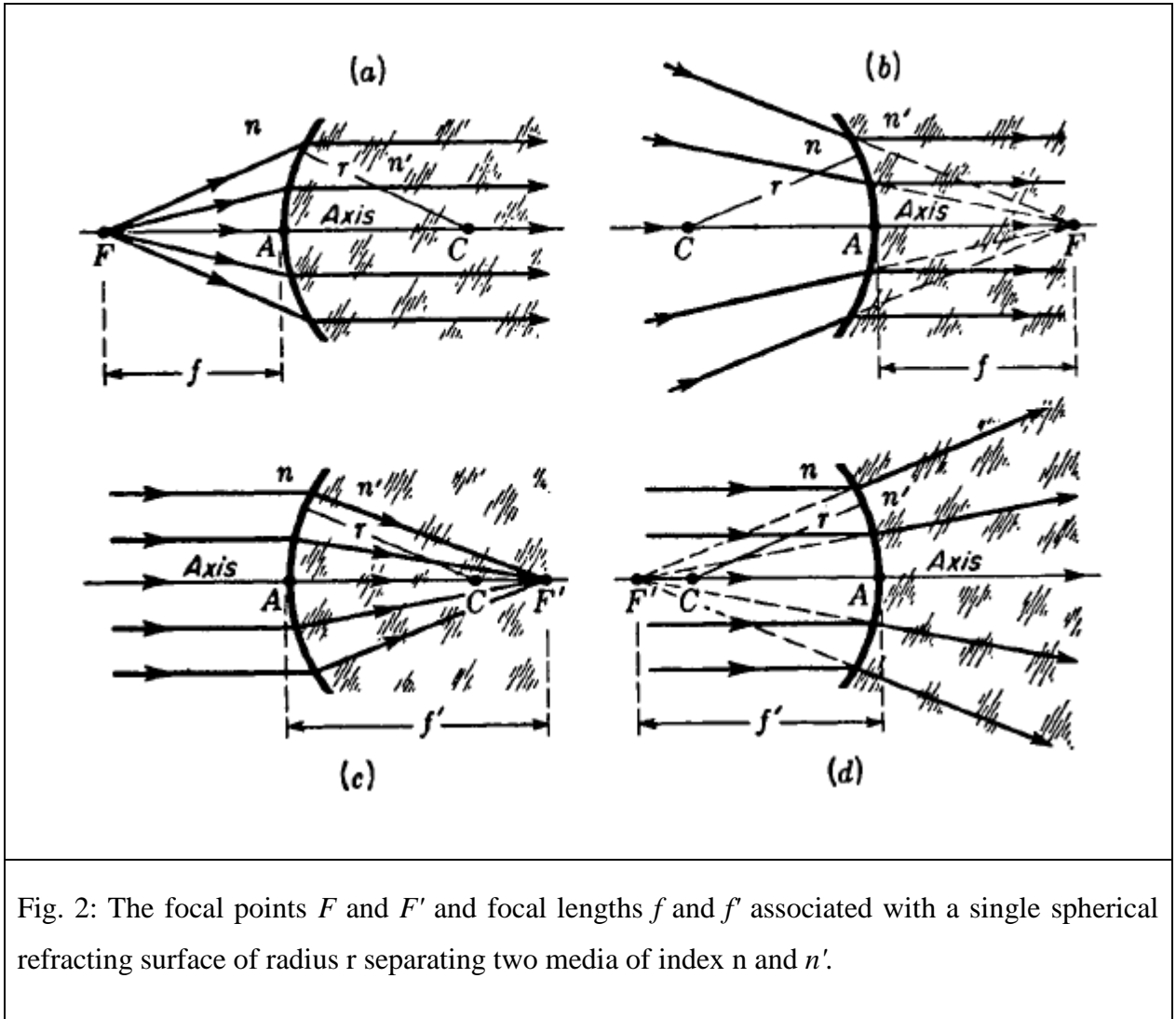


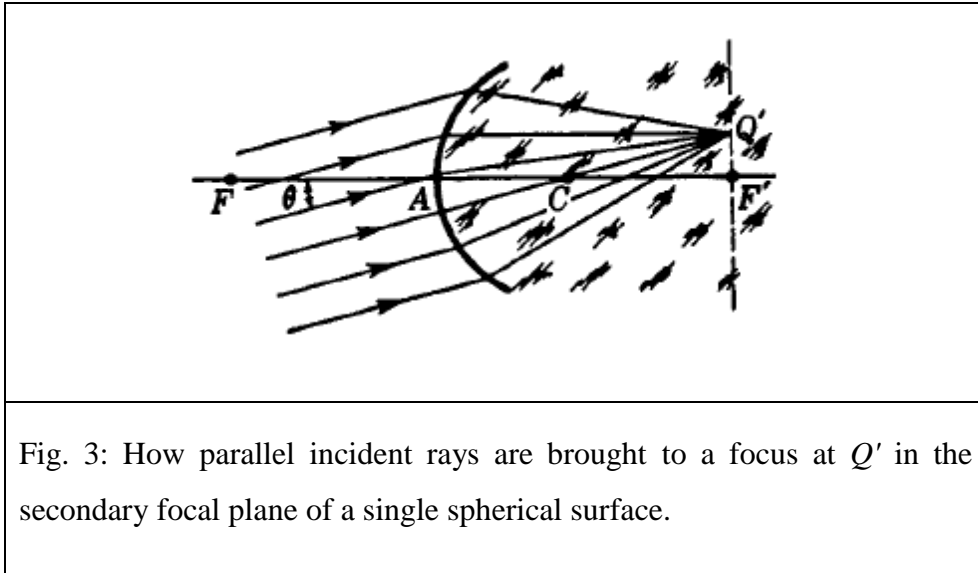
Fig. 2: The focal points F and F' and focal lengths f and f' associated with a single spherical refracting surface of radius r separating two media of index n and n' .

In optical diagrams it is common practice to show incident light rays traveling from left to right. A convex surface therefore is one in which the center of curvature C lies to the right of the vertex, while a concave surface is one in which C lies to the left of the vertex.

3. IMAGE FORMATION

A diagram illustrating image formation by a single refracting surface is given in Fig. 3. It has been drawn for the case in which the first medium is air with an index $n = 1$ and the second medium is glass with an index $n' = 1.60$. The focal lengths f and f' therefore have the ratio 1: 1.60. Experimentally it is observed that if the object is moved closer to the primary focal plane, the image will be formed farther to the right away from F' and will be larger, i.e., magnified. If the object is moved to the left, farther away from F , the image will be found closer to F' and will be smaller in size. All rays coming from the object point Q are shown brought to a focus at Q' .

This ideal condition never holds exactly for any actual case because of 1) light scattering 2) aberration 3) diffraction



4. VIRTUAL IMAGES

The image $M'Q'$ in Fig. 4 is a real image in the sense that if a flat screen is located there, a sharply defined image of the object MQ will be formed on the screen. Not all images, however, can be formed on a screen, as is illustrated in Fig. 5. Light rays from an object point Q are shown refracted by a concave spherical surface separating the two media of index $n = 1.0$ and $n' = 1.50$, respectively. The focal lengths have the ratio 1: 1.50.

Since the refracted rays are diverging, they will not come to a focus at any point. To an observer's eye located at the right, however, such rays will appear to be coming from the common point Q' . In other words, Q' is the image point corresponding to the object point Q . Similarly, M' is the image point corresponding to the object point M . Since the refracted rays do not come from Q' but only appear to do so, no image can be formed on a screen placed at M' . For this reason, such an image is said to be *virtual*.

5. CONJUGATE POINTS AND PLANES

The principle of the reversibility of light rays has the consequence that if $Q'M'$ in Fig. 4 were an object, an image would be formed at QM . Hence, if any object is placed at the position previously occupied by its image, it will be imaged at the position previously occupied by the object. The object and image are thus interchangeable, or conjugate. Any pair of object and image points such as M and M' in Fig. 4 are called *conjugate points*, and planes through these points perpendicular to the axis are called *conjugate planes*.

If one is given the radius of curvature r of a spherical surface separating two media of index n and n' , respectively, as well as the position of an object, there are three general methods that may be employed to determine the position and size of the image: (1) graphical methods, (2) experiment, and (3) calculation using the formula

$$\frac{n}{s} + \frac{n'}{s'} = \frac{n' - n}{r}$$

1

In this equation s is the object distance and s' is the image distance. This equation, called the *Gaussian formula for a single spherical surface*,

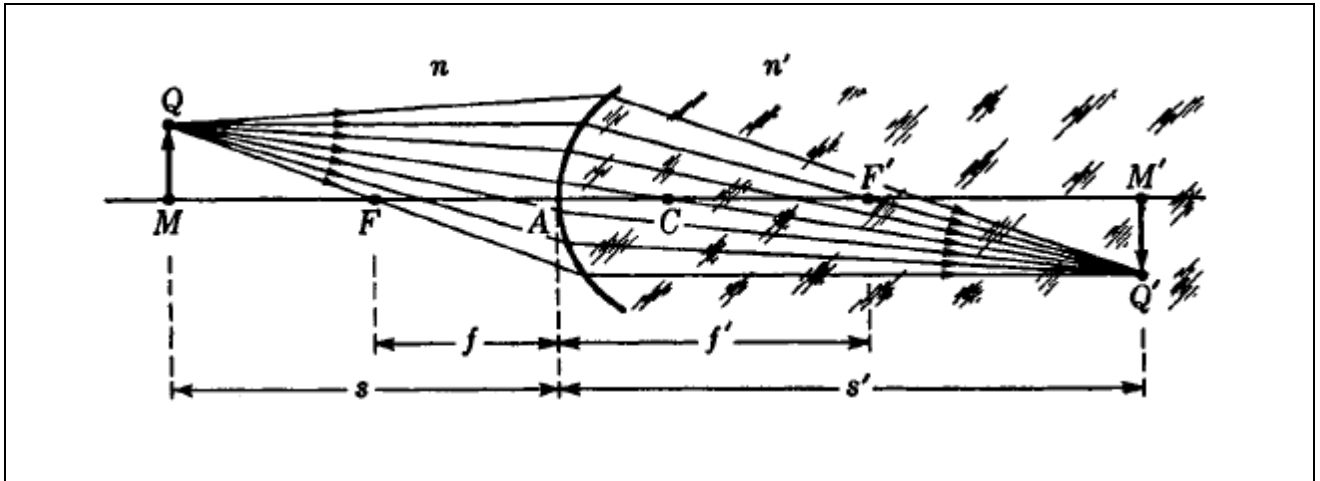


Fig. 4: All rays leaving the object point Q and passing through the refracting surface are brought to a focus at the image point Q' .

As an object M is brought closer to the primary focal point, Eq. (1) shows that the distance AM' of the image from the vertex becomes steadily greater and that in the limit when the object reaches F the refracted rays are parallel and the image is formed at infinity. Then we have $s' = \infty$, and Eq. (1) becomes

$$\frac{n}{s} + \frac{n'}{\infty} = \frac{n' - n}{r}$$

2

Since this particular object distance is called the primary focal length f , we may write:

$$\frac{n}{f} = \frac{n' - n}{r}$$

3

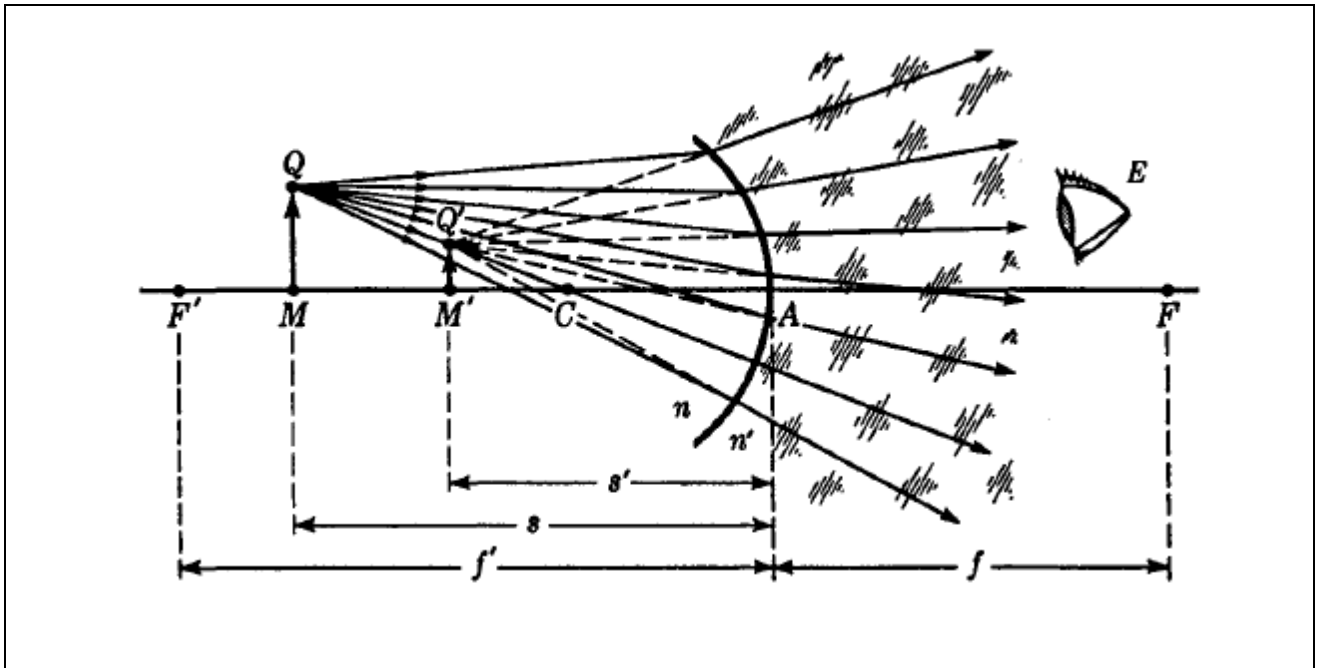


Fig. 5: All rays leaving the object point Q , and passing through the refracting surface appear to be coming from the virtual image point Q' .

Similarly, if the object distance is made larger and eventually approaches infinity, the image distance diminishes and becomes equal to f' in the limit, $s = \infty$. Then

$\frac{n}{\infty} + \frac{n'}{s'} = \frac{n' - n}{r}$	4
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since this value of s' represents the secondary focal length f' ,

$\frac{n'}{f'} = \frac{n' - n}{r}$	5
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Equating the left-hand members of Eqs. (3) and (5), we obtain

$\frac{n}{f} = \frac{n'}{f'} \quad \text{or} \quad \frac{n'}{n} = \frac{f'}{f}$	6
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When $(n' - n)/r$ in Eq. (1) is replaced by n/f or by n'/f' according to Eqs. (3) and (5), there results

$\frac{n}{s} + \frac{n'}{s'} = \frac{n}{f} \quad \text{or} \quad \frac{n}{s} + \frac{n'}{s'} = \frac{n'}{f'}$	7
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Both these equations give the conjugate distances for a single spherical surface.

6. CONVENTION OF SIGNS

1. All figures are drawn with the light traveling from left to right.
2. All object distances (s) are considered positive when they are measured to the left of the vertex and negative when they are measured to the right.
3. All image distances (s') are positive when they are measured to the right of the vertex and negative when to the left.
4. Both focal lengths are positive for a converging system and negative for a diverging system.
5. Object and image dimensions are positive when measured upward from the axis and negative when measured downward.
6. All convex surfaces are taken as having a positive radius, and all concave surfaces are taken as having a negative radius.