1. REFRACTION BY A PRISM

In a prism the two surfaces are inclined at some angle ex so that the deviation produced by the first surface is not annulled by the second but is further increased. The chromatic dispersion is also increased, and this is usually the main function of a prism. First let us consider, however, the geometrical optics of the prism for light of a single color, i.e., for *monochromatic light* such as is obtained from a sodium arc.

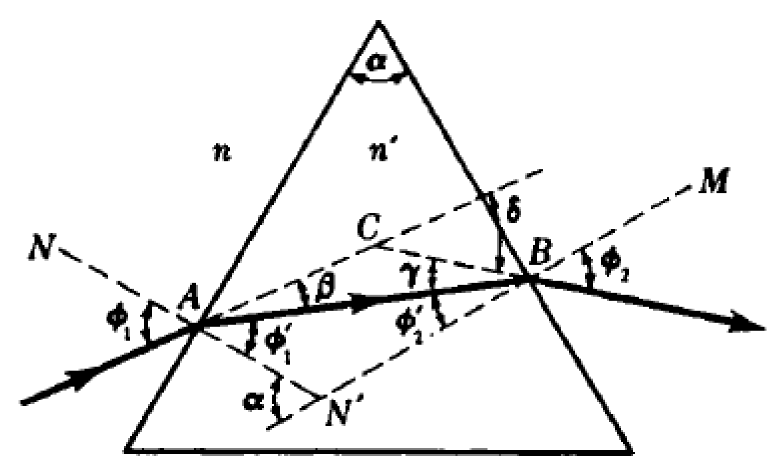

Fig. 1: The geometry associated with refraction by a prism.

The solid ray in Fig. 1 shows the path of a ray incident on the first surface at the angle ϕ_1. Its refraction at the second surface, as well as at the first surface, obeys Snell's law, so that in terms of the angles shown

$\frac{\sin \emptyset_1}{\sin \emptyset_1^{\backslash}} = \frac{n^{\backslash}}{n} = \frac{\sin \emptyset_2}{\sin \emptyset_2^{\backslash}}$	1

The angle of deviation produced by the first surface is $\beta = \emptyset_1 - \emptyset_1^{\backslash}$, and that produced by the second surface is $\gamma = \emptyset_2 - \emptyset_2^{\backslash}$. The total angle of deviation between the incident and emergent rays is given by

$\delta = \beta + \gamma$	2

Since *NN'* and *MN'* are perpendicular to the two prism faces, α is also the angle at *N'*. From triangle *ABN'* and the exterior angle α, we obtain

$\alpha = \emptyset_1^{\backslash} + \emptyset_2^{\backslash}$	3

Combining the above equations, we obtained

$\delta = \beta + \gamma = \emptyset_1 - \emptyset_1^{\backslash} + \emptyset_2 - \emptyset_2^{\backslash} = \emptyset_1 + \emptyset_2 - \left(\emptyset_1^{\backslash} + \emptyset_2^{\backslash}\right) = \emptyset_1 + \emptyset_2 - \alpha$	4

2. MINIMUM DEVIATION

When the total angle of deviation δ for any given prism is calculated by the use of the above equations, it is found to vary considerably with the angle of incidence. The smallest deviation angle, called the *angle of minimum deviation* δ occurs at that particular angle of incidence where the refracted ray inside the prism makes equal angles with the two prism faces (see Fig. 2). In this special case

$\emptyset_1 = \emptyset_2 , \emptyset_1^{\backslash} = \emptyset_2^{\backslash} , \beta = \gamma$	5

To prove these angles equal, assume ϕ_1 does not equal ϕ_2 when minimum deviation occurs. In the triangle *ABC* in Fig.2 the exterior angle δ_m equals the sum of the opposite interior angles $\beta + \gamma$. Similarly, for the triangle *ABN'*, the exterior angle α equals the sum $\emptyset_1^{\backslash} + \emptyset_2^{\backslash}$ Consequently.

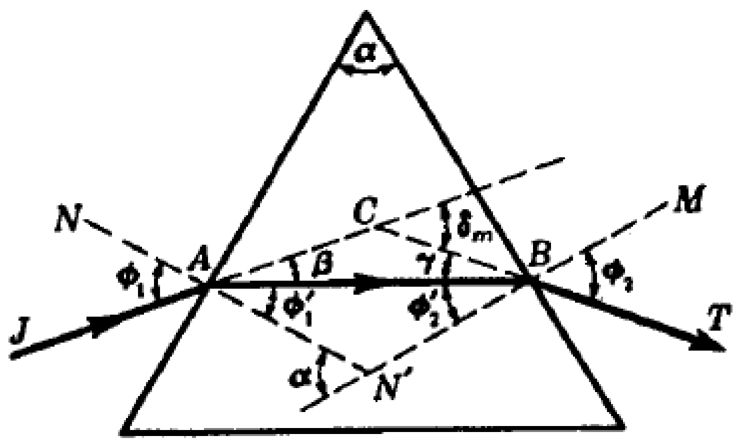

Fig. 2: The geometry of a light ray traversing a prism at minimum deviation.

$\alpha = 2\emptyset_1^{\backslash} \quad , \ \delta_m = 2\beta \, , \quad \emptyset_1 = \emptyset_1^{\backslash} + \beta$	6

Solving these equations for $\emptyset_1^{\backslash}$ and $\emptyset_1$ gives

$\emptyset_1^{\backslash} = \frac{1}{2}\,\alpha \quad , \ \emptyset_1 = \frac{1}{2}\ (\alpha + \delta_m)$	7

Since by Snell's law $n^{\backslash}/n = {\sin \emptyset_1}/{\sin \emptyset_1^{\backslash}}$

$\frac{n^{\backslash}}{n} = \frac{\sin\frac{1}{2}\ (\alpha + \delta_m)}{\sin\frac{1}{2}\ \alpha}$	8

The most accurate measurements of refractive index are made by placing the sample in the form of a prism on the table of a spectrometer and measuring the angles (δ_m, α) for each color desired.

3. THIN PRISMS

The equations for the prism become much simpler when the refracting angle (α) becomes small enough to ensure that its sine and the sine of the angle of deviation (δ) may be set equal to the angles themselves. Even at an angle of 0.1 rad, or 5.7°, the difference between the angle and its sine is less than 0.2 percent. For prisms having a refracting angle of only a few degrees, we can therefore simplify Eq. (8) by writing

$\frac{n^{\backslash}}{n} = \frac{\sin\frac{1}{2}(\alpha + \delta_m)}{\sin\frac{1}{2}\alpha} = \frac{\delta_m + \alpha}{\alpha} \qquad and\ \delta = \left(n^{\backslash} - 1\right)\alpha$ Thin prism in air	9

The subscript on (δ) has been dropped because such prisms are always used at or near minimum deviation, and (n) has been dropped because it will be assumed that the surrounding medium is air, n = 1. It is customary to measure the power of a prism by the deflection of the ray in centimeters at a distance of 1 m, in which case the unit of power is called *the prism diopter (D)*. A prism having a power of 1 diopter therefore displaces the ray on a screen 1 m away by 1 cm.

4. REFLECTION OF DIVERGENT RAYS

When a divergent pencil of light is reflected at a plane surface, it remains divergent. All rays originating from a point Q (Fig. 3) will after reflection appear to come from another point Q' symmetrically placed behind the mirror. The proof of this proposition follows at once from the application of the law of reflection, according to which all the angles labeled (ϕ) in the figure must be equal. Under these conditions the distances QA and AQ' along the line QAQ' drawn perpendicular to the surface must be equal; i.e.,

$S = S^{\backslash}$, object distance = image distance

The point Q' is said to be a virtual image of Q since when the eye receives the reflected rays, they appear to come from a source at Q' but do not actually pass through Q', as would be the case if it were a real image.

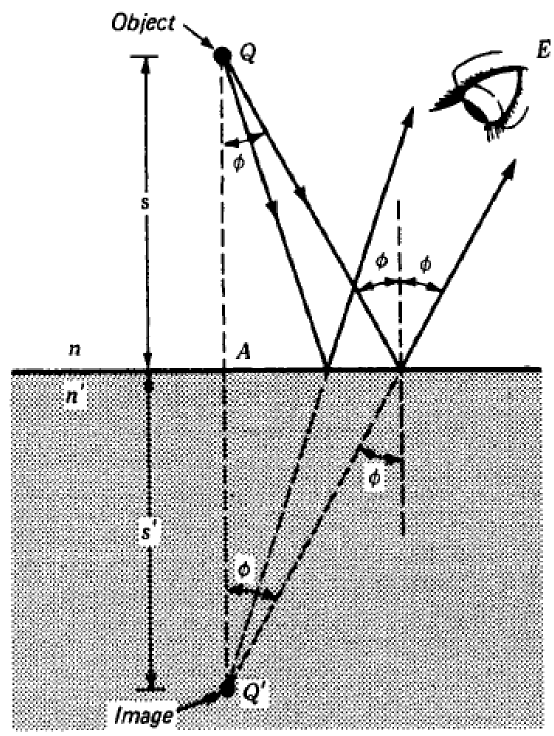

Fig. 3: The reflection of divergent rays of light from a plane surface.

5. REFRACTION OF DIVERGENT RAYS

If an object is embedded in clear glass or plastic or is immersed in a transparent liquid such as water, the image appears closer to the surface. Fig. 4 has been drawn accurately to scale for an object Q located in water of index 1.3330 at a depth (s) below the surface. Light rays diverging from this object arrive at the surface at angles (ϕ). There they are refracted at larger angles (ϕ') as shown. Extending these emergent rays backward, we locate their intersections in pairs. These are image points, or virtual images. As the observer changes his position, the virtual image moves closer to the surface and along the curve formed by the successive images.

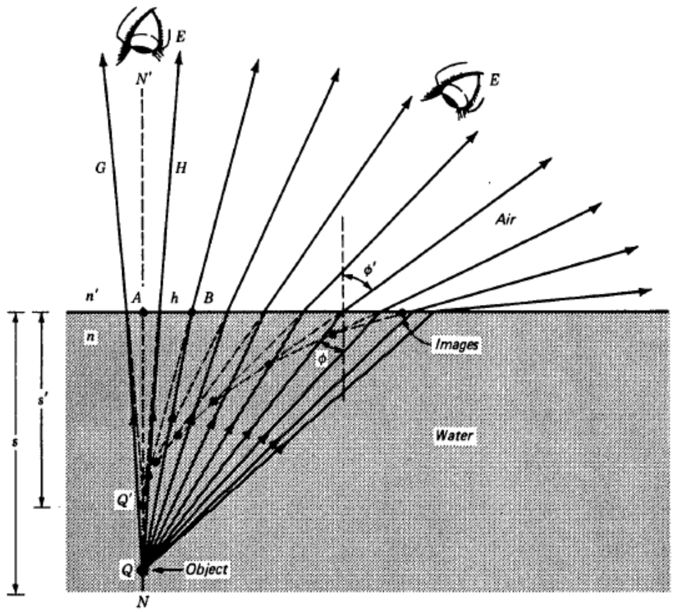

Fig. 4: Image positions of an object under water as seen by an observer above; $n > n'$.

If the object is located in the less dense medium and is observed from the medium of higher index, we obtain an entirely different view (see Fig. 5). An object Q in air is observed by an underwater swimmer or fish. Rays of light diverging from any point of this object are refracted according to Snell's law. Extended backward to their intersections, their virtual images are located. Note how far away these images are for large angles of ϕ and ϕ'.

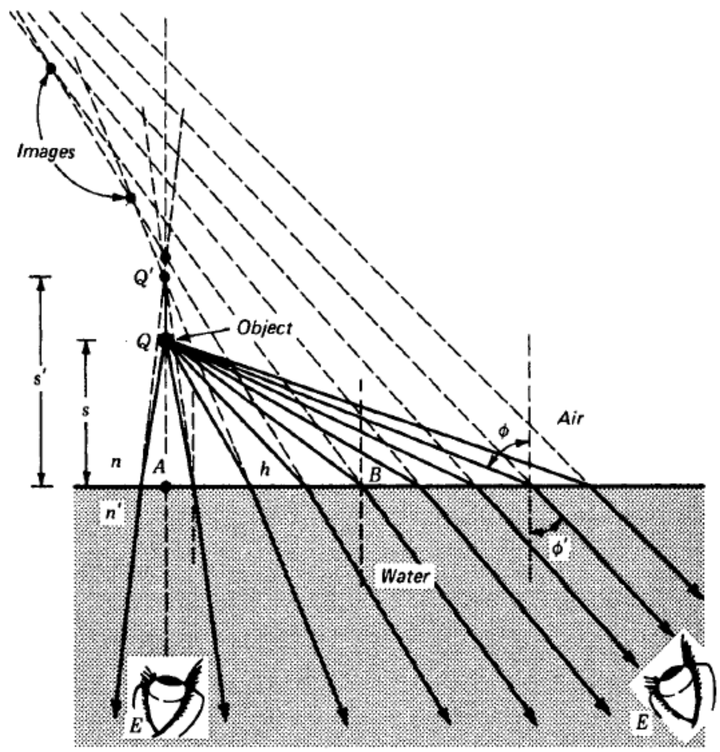

Fig. 5: Image positions of an object in air as seen by an observer under water; $n < n'$.

6. IMAGES FORMED BY PARAXIAL RAYS

Of particular interest to many observers are the object and image distances s and s' for rays making small angles ϕ and ϕ'. *Paraxial rays: Rays for which angles are small enough to permit setting the cosines equal to unity and the sines and tangents equal to the angles.*

Consider the right triangles QAB and Q'AB in Fig. 4, redrawn in Fig. 6. Since there is a common side AB = h, we can write

$h = s \tan \phi = s' \tan \phi'$

From this we find

$$s' = s\,\frac{\tan \phi}{\tan \phi'} = s\,\frac{\sin \phi \cos \phi'}{\cos \phi \sin \phi'} \tag{1}$$

Applying Snell's law,

$$\frac{\sin \phi}{\sin \phi'} = \frac{n'}{n} \tag{2}$$

we obtain on substitution in Eq. (2)

$$s' = s\,\frac{n'}{n}\frac{\cos \phi'}{\cos \phi} \tag{3}$$

For paraxial rays like the ones shown in the diagram, angles ϕ and ϕ' are very small; Eq. (1) can be written

$$s' = s\,\frac{\phi}{\phi'}\ , \qquad or \qquad \frac{s'}{s} = \frac{\phi}{\phi'} \tag{4}$$

and Eq. (3) written

$$\frac{\phi}{\phi'} = \frac{n'}{n} \tag{5}$$

Together Eqs. (4) and (5) provide the simple relation

$\frac{s}{s^{\backslash}} = \frac{n^{\backslash}}{n} \quad Paraxial\ rays$	6

The ratio of the image to object distance for paraxial rays is just equal to the ratio of the indices of refraction.

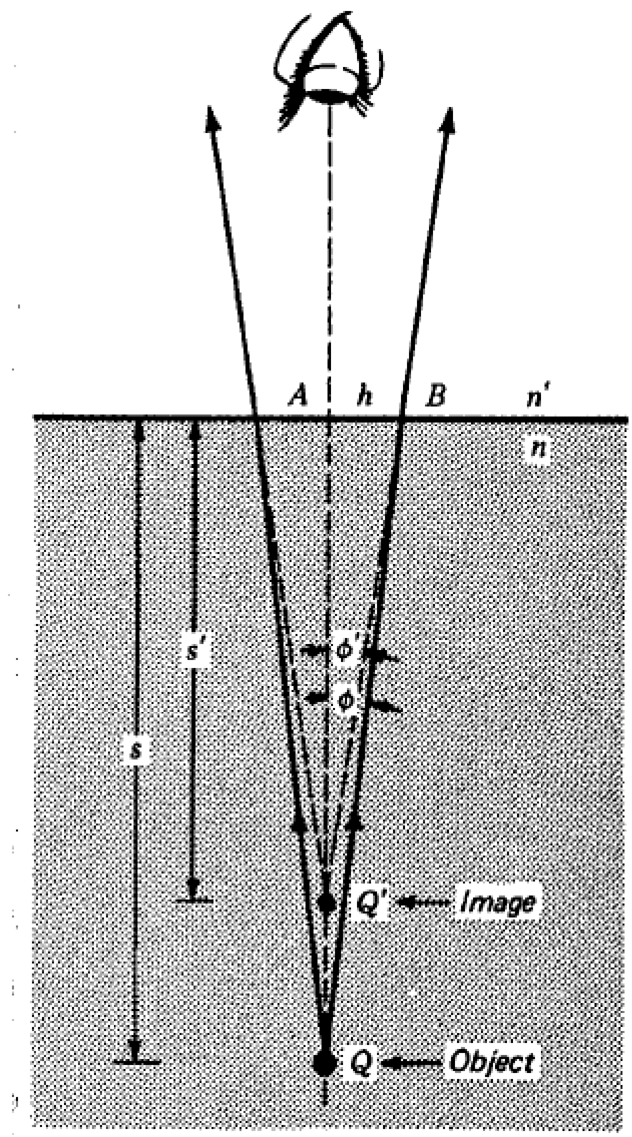

Fig. 6: Paraxial rays for an object in water and observed from the air above.