1. THE CRITICAL ANGLE AND TOTAL REFLECTION

light passes from one medium like air into another medium like glass or water the angle of refraction is always less than the angle of incidence. While a decrease in angle occurs for all angles of incidence, there exists a range of refracted angles for which no refracted light is possible. A diagram illustrating this principle is shown in Fig. 1, where for several angles of incidence, from 0 to 90°, the corresponding angles of refraction are shown from 0° to θ_c.

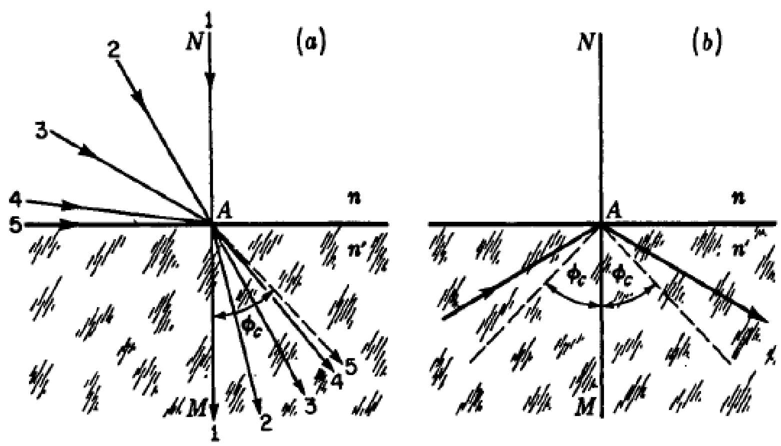

Fig. 1: Refraction and total reflection: *(a)* the critical angle is the limiting angle of refraction; *(b)* total reflection beyond the critical angle.

It will be seen that in the limiting case, where the incident rays approach an angle of 90° with the normal, the refracted rays approach a fixed angle ϕ_c beyond which no refracted light is possible. This particular angle ϕ_c, for which ϕ = 90°, is called the *critical angle.*

A formula for calculating the critical angle is obtained by substituting ϕ = 90°, or sin ϕ = I, in Snell's law

$n\,x\,1 = n^{\backslash} \sin\emptyset_c \quad , \sin\emptyset_c = \frac{1}{n^{\backslash}}$	1

The critical angle for the boundary separating two optical media is defined as the smallest angle of incidence, in the medium of greater index, for which light is totally reflected.

Total reflection is really total in the sense that no energy is lost upon reflection. In any device intended to utilize this property there will, however, be small losses due to absorption in the medium and to reflections at the surfaces where the light enters and leaves the medium. The commonest devices of this kind are called *total reflection prisms,* which are glass prisms with two angles of 45° and one of 90°. As shown in Fig. *2(a),* the light usually enters perpendicular to one of the shorter faces, is totally reflected from the hypotenuse, and leaves at right angles to the other short face. This deviates the rays through a right angle. Such a prism may also be used in two other ways which are illustrated in *(b)* and *(c)* of the figure. The Dove prism *(c)* interchanges the two rays, and if the prism is rotated about the direction of the light, they rotate around each other with twice the angular velocity of the prism.

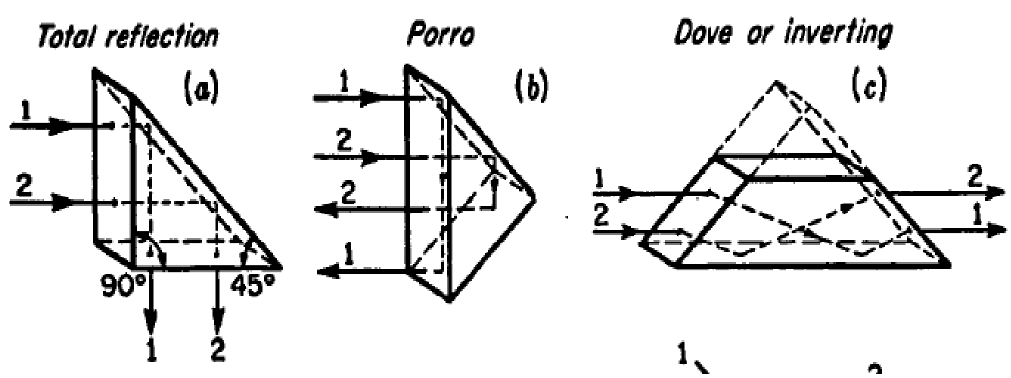

Fig. 2: Reflecting prisms utilizing the principle of total reflection.

A Dove prism is a type of reflective prism which is used to invert an image. Dove prisms are shaped from a truncated right-angle prism. The Dove prism is named for its inventor, Heinrich Wilhelm Dove. A beam of light travelling parallel to the longitudinal axis, entering one of the sloped faces of the prism undergoes total internal reflection from the inside of the longest (bottom) face and emerges from the opposite sloped face. Images passing through the prism are flipped (mirrored), and because only one reflection takes place, the image is also inverted but not laterally transposed.

2. PLANE-PARALLEL PLATE

When a single ray traverses a glass plate with plane surfaces that are parallel to each other, it emerges parallel to its original direction but with a lateral displacement *d* which increases with the angle of incidence ϕ. Using the notation shown in Fig. 3, we may apply the law of refraction and some simple trigonometry to find the displacement *d*. Starting with the right triangle *ABE*, we can write

$d = l \sin(\emptyset - \emptyset^{\backslash})$	2

by the trigonometric relation for *the sine of the difference between two angles*,

$d = l \left(\sin\emptyset \cos\emptyset^{\backslash} - \sin\emptyset^{\backslash} \cos\emptyset\right)$	3

From the right triangle *ABC* we can write

$l = \frac{t}{\cos\emptyset^{\backslash}}$	4

substituted in Eq. (3), gives

$d = t \left(\frac{\sin\emptyset \cos\emptyset^{\backslash}}{\cos\emptyset^{\backslash}} - \frac{\sin\emptyset^{\backslash} \cos\emptyset}{\cos\emptyset^{\backslash}}\right)$	5

From Snell's law $\sin\phi' = \frac{n}{n'}\sin\phi$

which upon substitution in Eq. (5), gives

$d = t \left(\sin\emptyset - \frac{\cos\emptyset}{\cos\emptyset^{\backslash}} \frac{n}{n^{\backslash}} \sin\emptyset\right)$	6

$d = t \sin\emptyset \left(1 - \frac{n}{n^{\backslash}} \frac{\cos\emptyset}{\cos\emptyset^{\backslash}}\right)$	7

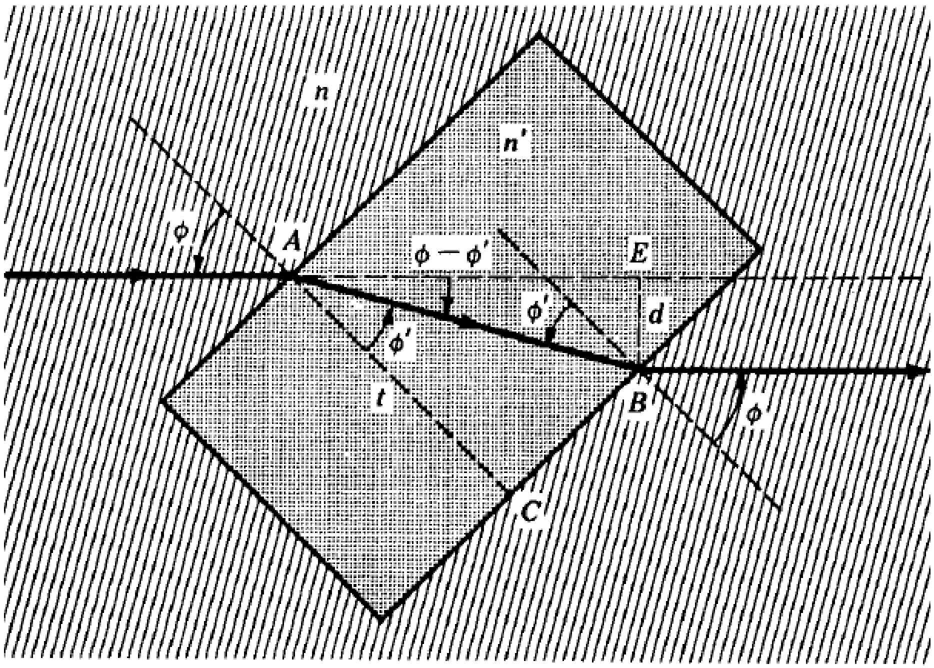

Fig. 3: Refraction by a plane-parallel plate.